ONLINE SUPPLEMENT

WAVELET ANALYSIS TECHNIQUES

The following section explains in more detail the wavelet based comparisons described within the signal analysis section.

The continuous wavelet transform $W_n^x(\text{eq. 1})$ describes the convolution of time series x_n with translated and scaled versions of a mother wavelet (ψ) where s denotes a scaling factor and η a translation factor¹². The complex Morlet wavelet (eq. 2) with a centre frequency (ω_0) 1 and scaling factor (η) 1 was used as it offers good localisation with respect to time and frequency. The continuous wavelet transform from scales 1 to 150 was performed with respect to time points (n) on 0.5Hz signal data using Matlab (Mathworks, USA) producing W_n^x representing complex time series of arterial blood pressure, TCD and NIRS signals. The wavelet cross transform (W_n^y) between these wavelet transforms (eq. 3) can be used to calculate measures of power and the instantaneous phase difference $\Delta \phi$ (eq. 4).

Analysis within the cross transform was confined to areas outside the cone of influence of edge effects. This is defined by the e folding time of the Morlet wavelet ($\sqrt{2s}$).

$$W_n^x(s) = \sqrt{\frac{\delta t}{s}} \sum_{n'=1}^N x_{n'} \psi_0 \left[(n'-n) \frac{\delta t}{s} \right]$$
(1)

$$\psi_0(\eta) = \pi^{-1/4} e^{i\omega_0 \eta} e^{-\frac{1}{2}\eta^2}$$
(2)

$$W_n^{xy} = W_n^x . W_n^y \tag{3}$$

$$\Delta \phi = \tan^{-1} \frac{\Im \left(W_n^{xy} \right)}{\Re \left(W_n^{xy} \right)} \tag{4}$$

$$S = \cos^{z}(\Delta\phi) \tag{5}$$

$$C_n^2(s) = \frac{\left| \left\langle W_n^{xy}(s) s^{-1} \right\rangle \right|^2}{\left\langle \left| W_n^{xx}(s) \right| s^{-1} \right\rangle \left\langle \left| W_n^{yy}(s) \right| s^{-1} \right\rangle}$$
(6)

The semblance (eq. 5)[•] was used as a measure of instantaneous phase difference. This makes sense intuitively as it creates an index of +1 when a wave is completely in phase and -1 when 180° out of phase a in an identical fashion to the PRx and Mx. Areas of particular interest at 1 and -1 may be more clearly defined by increasing the exponent z. Coherence is a measure of normalised signal power, indicating areas where signal power co-varies and as such reflects synchronisation of phase and change in power . Wavelet coherence was calculated (eq. 6) by the methods described by Torrence and Compo⁺ modified to calculate coefficients at each scale. The brackets denote smoothing with respect to scale and time which is of critical importance as coherence would otherwise equal 1 at all points. Smoothing was performed in a Gaussian distribution defined by the footprint of the Morlet wavelet at each scale. Statistical testing for areas of significant coherence based on Monte-carlo modelling was performed using algorithms described previously⁻. This involved performing 1000 wavelet coherence comparisons of red noise generated from the autocorrelation coefficient of constituent signals to define 95%

confidence boundaries at each scale. This allows specification of an area of time frequency space where significant linear relationships between input signals exists. An identical simulation was used to generate confidence intervals for the mean coherence of all patients.



Figure 4. Example signal analysis demonstrating coherence and semblance analysis of two example signals with added noise. From segment 1 to 3 the waves in A and B are synchronised and reduce in frequency in three steps. This is seen as dark grey areas of high coherence which are above the 95% confidence limit. During segment 1-2 the waves are in phase – hence have a semblance of 1 (red). In segment 3 waves are synchronised but have a phase difference of 180° thus a semblance of -1 (blue). In 4 there is random noise and hence no coherence or fixed pattern of semblance.

SUPPLEMENTARY RESULTS

The following section describes the NIRS, ICP and TCD slow wave activity in all 27 datasets using multi-panel wavelet plots of semblance and coherence as demonstrated in Fig. 3.1, 3.2 and 4.

In 12/27 cases either Mx or PRx was greater than 0.3, defining impaired reactivity, these datasets are displayed in fig. 5.1. The remainder are shown in fig. 5.2. It can be seen that in general coherence between ICP/THI and TCD/rSO₂ is not consistently present in the slow wave spectrum (<0.05 Hz). This relationship between NIRS, TCD and ICP varies with both time and frequency and in many cases no clear relationship is visible.

Several key features can be seen examining the semblance plots: 1) In cases with impaired reactivity semblance tends more towards 1 (below 0.05Hz), indicating pressure passive oscillations of TCD, ICP, rSO₂ and THI. However this is less pronounced in the case of rSO₂ and THI. 2) In intact reactivity (PRx <0.3, Mx <0.3) there is dynamic variation

in phase between arterial blood pressure and TCD, ICP, rSO₂ and THI in the region <0.05Hz (fig. 5.2).



Figure 5.1 Wavelet slow wave analysis of cases with impaired reactivity. This figure is a multi-panel wavelet plot of all cases with impaired reactivity (PRx >0.3 or Mx >0.3). The panels are identical to those shown in the preceding wavelet figures, including measures of semblance and coherence. Each patient is shown as a horizontal row. Slow wave activity is represented by the majority of the area of each panel (<0.05Hz). Semblance between ABP and neuromonitoring is shown in blue-red. Coherence is in grey with the 95% significance threshold indicated as the black contour. It can be seen that there is a predominance of semblance tending towards 1 (red) in the slow wave spectrum indicating passive entrainment of TCD, ICP, rSO₂ and THI as would be expected with impaired reactivity. Importantly in many cases the response of THI and rSO₂ is less consistent (less red). Likewise coherence between ICP/THI and TCD/ rSO₂ is highly variable in the time and frequency domains. In many datasets there is little coherence <0.05Hz.



Figure 5.2. Wavelet slow wave analysis of cases with intact reactivity. This is a multipanel wavelet plot of all cases with intact reactivity (PRx <0.3, Mx <0.3). In comparison to fig. 5.1 these demonstrate much more variable semblance indicated by greater incidence of semblance of -1 (blue) with time and frequency. While coherence between TCD/ rSO₂ and ICP/THI is still present in the majority of datasets, it is highly variable in both time and frequency domains.

REFERENCES

- 1. Grinsted A, Moore JC, Jevrejeva S. Application of the cross wavelet transform and wavelet coherence to geophysical time series. Nonlinear Processes in Geophysics 2004;11:561–6.
- 2. Daubechies I. The wavelet transform, time-frequency localization and signal analysis. IEEE Trans Inform Theory 1990;36:961–1005.
- 3. Cooper GRJ, Cowan DR. Comparing time series using wavelet-based semblance analysis. Computers & Geosciences 2008;34:95–102.
- 4. Torrence C, Compo GP. A Practical Guide to Wavelet Analysis. Bulletin of the American Meteorological Society 1998;79:61–78.