**Updates to the QE model since 2009**

*Change to quality scoring*

Consider a collection of *k* independent studies, the *j*th of which has estimated effect size  which varies from its true effect size, through random error. Also consider that the true effects, , also vary from an underlying common effect, , through bias. This bias would include the possibility of some diversity of true effects (which remain similar) across studies (in which case  would simply be the mean of the true (unbiased) effects). A greater diversity that leads to dissimilarity of effects would not be meta-analysed 1.

For the QE model, the weighted estimator, , has weights that are adjusted from inverse variance weights based on the quality scores, , derived from each study and which are expressed on a scale between 0 and 1 by dividing by the maximum possible under the scale. We interpret the rescaled quality of each individual study as an intra-class correlation () depicting the proportion of total bias variance () not related to variance from internal study bias 2. This value of , is thus is given by and can be expressed relative to the maximum () in the list of studies. Creating such a relative score serves the purpose of assigning a maximum value of 1 to the best study and proportionally less to lower scoring studies. Mathematically, the relative quality score rescaled from 1 downwards serves as a proportional indicator of decreasing relative quality rank, say , and this can then be defined by the following expression:

 [1]

Expression [1] indicates that always starts from 1 since the best study has  and has a minimum value of zero since. If we consider that errors due to chance are of equal importance to errors due to bias 3, then can be used to discount the *j*th study’s inverse variance weight. Thus, the inverse variance weight remains unaltered when  and reduces proportionally as decreases towards zero, and at or close to zero the study is effectively excluded from the meta-analysis. This new weight is given by:

 [2]

To generate a weight that is influenced by both chance and bias thus requires weights to drop from to due to the influence of variance due to bias. An important aspect to keep in mind is that there is no attempt here at bias quantification and there is no connection between a quality score and the magnitude or direction of change in an effect size. This model utilizes the fact that variance due to bias can be modeled through a quality score.

The decrease in the weight defined by expression [2] (from inverse variance weight) in each study will therefore be by an amount given by:

 [3]

To reduce the inherent bias in this weighting scheme, we can pool the weight decrement across studies using expression [3], and this is given by , which then can be split into *k* parts whose size is *proportional to * and added to each study’s weight so that the sum of weights remains unchanged from the sum of the inverse variance weights. This adjustment was proposed in 2009 4. The advantage gained from doing this additional procedure is a bias reduction for the QE estimator. This quantity called , is then used to compute the final QE model weight  for each study given by:

 [4]

and the weights that sum to 1 are given by



While the weights in expression [4] will not equal the expected value of the unknown weight which is , the relative distribution should now follow that of the expected weight to a large extent. Since we are only interested in relative weights (that sum to 1) this works quite well to decrease estimator variance beyond that achievable through inverse variance weights alone.

*Model variance*

The theoretical model variance is computed by modeling overdispersion through a quasi-likelihood approach 5,6. This implies that the meta-analysis is performed under a fixed effect assumption  where  is a moment-based estimate of the between-studies variance proposed by DerSimonian and Laird 7 and the variance of the estimator inflated to account for the heterogeneity, thus preventing a reduction in coverage. This has the advantage of being based purely on the variance-to-mean relationship (rather than on distributional assumptions) with variance appropriately inflated using a scale parameter, . The latter can be defined by interpreting the multiplicative factor as an intra-class correlation (ICC) as described by Kulinskaya & Olkin 8 where the and the scale parameter is defined as:

 [5]

The variance of any weighted estimator is given by  where  is any series of weights that sum to 1. The latter is then inflated to . Using the latter expression, the variance of the estimator under the the QE model weights then reduces to:

 [6]

**Detailed simulation method**

We denote Ln(odds ratio), , to be a true effect size, the number of patients in the *J*th study to be , and the number of studies in each meta-analysis to be *k.* These are then generated randomly from 45 combinations of three distributions and 15 possible numbers of studies (5 to 19; *k*). The three distributions of  are an extremely skewed Delaporte distribution and two uniform distributions with parameters as follows:



 The Delaporte distribution mimics meta-analyses which have mostly uniformly small sample sizes (below 200) but the occasional mega-trial (up to the largest sample size of 58,000). Total survivals  and deaths  are then determined by allocating a uniform proportion () between 0.820 and 0.993 out of  as the number that survived, , and thus the number of non-survivals was . The survivors () are then distributed between treated () and untreated () groups by making this allocation in a uniform proportion between 0.464 and 0.545 to the treated group and the rest allocated to the untreated control group. The numbers of treated and untreated survivors were then used as parameters in a beta distribution to generate the proportion of survivors,  , who were treated. The latter was then used to compute a similar proportion, , of non-survivors who were treated such that it resulted in the true OR of  which is computed as:



The proportions  and were then used to create the four-fold cell counts and these were then used to generate the sampling variance,as follows:



A synthetic bias variance  was generated next using (ranging from 0 to 9 in steps of 1) as a starting (average) value for synthetic bias variance where  isa “quality ICC” 2 for each study which re-distributes the starting bias variance to studies. The value of *m²* in each run was constant and as the run sequence increased the value of *m²* increased (= run sequence - 1) to reflect heterogeneity.

The final study estimate, , was generated from a normal distribution with mean  and variance . The final four-fold cell counts were generated by maintaining the proportion  of treated survivors but re-computing a new proportion of treated non-survivors,, that would equal the effect  where:



These proportions (and) were used to compute the numbers of treated non-survivors and treated survivors denoted respectively where subscript 1 denotes non-survival and subscript 2 denotes survival in the *j*th study. From this the final four-fold cell counts could be reconstructed to arrive at the variance of the study effect estimates as follows:

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The relative rank of the study quality is given by  which is an input into the QE method and in this simulation three different variants of the rank input to the QE method were assessed as follows:



where unrelated to . The uncertain approach represents the generation of in real life from  derived from the data, the latter being given by

,

and is clearly an uncertain process in real life represented in this simulation by the beta distribution around . We also simulated what happens when was generated from completely random values of  unrelated to the actual values used in the simulation. Finally, we simulated no rank input by setting all  to 1 to represent equal ranking of the studies irrespective of the  value in the simulation (we call this the IVhet method in MetaXL).

The final pooled estimate across all studies were then obtained using MetaXL (see below) via the four estimators (three variants of the QE estimator and the RE estimator) and this was repeated over 10,000 iterations from which performance measures were computed as described by Burton et al 9. Each simulation consisted of 10 runs with increasing starting bias variance (= run number -1) and as the latter increased, heterogeneity progressively increased. The degree of heterogeneity was indicated by the median value of the method of moments between study variance 7 ,  , in each run. For each estimator, ten simulations (total 100 runs) were performed each with a different (ln(OR) 0.5 to 2.3 in steps of 0.2) using *Ersatz* version 1.31 and all meta-analysis model pooled estimates were automatically computed using *MetaXL* version 2.0, both from Epigear International Pty Ltd, Brisbane, Australia ([www.epigear.com](http://www.epigear.com)). The simulation sheet is available from us on request but requires both MetaXL and Ersatz installed to run.

**SIMULATION RESULTS FOR OTHER EFFECT SIZES**

**Ln (0.5)**

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Tau Sq | IVhet MSE | RE MSE | QE MSE | QE-r MSE |  | IVhet var | RE var | QE var | QE-r var |  | IVhet bias2 | RE bias2 | QE bias2 | QE-r bias2 |
| 0.000 | 0.030 | 0.031 | 0.049 | 0.049 |  | 0.027 | 0.029 | 0.048 | 0.048 |  | 0.003 | 0.003 | 0.001 | 0.001 |
| 0.689 | 0.227 | 0.185 | 0.122 | 0.239 |  | 0.187 | 0.175 | 0.114 | 0.225 |  | 0.040 | 0.010 | 0.007 | 0.013 |
| 1.209 | 0.325 | 0.323 | 0.177 | 0.377 |  | 0.262 | 0.315 | 0.165 | 0.358 |  | 0.062 | 0.009 | 0.012 | 0.019 |
| 1.520 | 0.394 | 0.452 | 0.227 | 0.501 |  | 0.319 | 0.443 | 0.211 | 0.479 |  | 0.074 | 0.009 | 0.016 | 0.023 |
| 1.845 | 0.456 | 0.606 | 0.279 | 0.617 |  | 0.365 | 0.597 | 0.256 | 0.590 |  | 0.091 | 0.009 | 0.023 | 0.028 |
| 2.107 | 0.490 | 0.736 | 0.304 | 0.695 |  | 0.402 | 0.729 | 0.283 | 0.666 |  | 0.088 | 0.008 | 0.021 | 0.030 |
| 2.434 | 0.566 | 0.886 | 0.357 | 0.820 |  | 0.464 | 0.878 | 0.330 | 0.791 |  | 0.102 | 0.007 | 0.027 | 0.029 |
| 2.684 | 0.622 | 1.013 | 0.399 | 0.948 |  | 0.505 | 1.004 | 0.365 | 0.911 |  | 0.118 | 0.009 | 0.034 | 0.037 |
| 2.955 | 0.662 | 1.183 | 0.422 | 1.034 |  | 0.536 | 1.176 | 0.385 | 0.995 |  | 0.126 | 0.007 | 0.037 | 0.039 |
| 3.205 | 0.693 | 1.274 | 0.453 | 1.119 |  | 0.568 | 1.266 | 0.414 | 1.076 |  | 0.125 | 0.008 | 0.039 | 0.043 |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Tau sq | RE coverage | RE width | QE coverage | QE width | IVhet coverage | IVhet width | QE-r coverage | QE-r width |
| 0.000 | 0.954 | 0.529 | 0.965 | 0.710 | 0.956 | 0.546 | 0.963 | 0.707 |
| 0.689 | 0.870 | 1.331 | 0.966 | 1.682 | 0.935 | 1.747 | 0.917 | 1.687 |
| 1.209 | 0.848 | 1.668 | 0.973 | 2.155 | 0.948 | 2.279 | 0.912 | 2.129 |
| 1.520 | 0.829 | 1.867 | 0.966 | 2.445 | 0.945 | 2.599 | 0.901 | 2.405 |
| 1.845 | 0.809 | 2.052 | 0.967 | 2.730 | 0.949 | 2.899 | 0.891 | 2.651 |
| 2.107 | 0.798 | 2.185 | 0.970 | 2.943 | 0.956 | 3.141 | 0.892 | 2.847 |
| 2.434 | 0.794 | 2.373 | 0.971 | 3.219 | 0.961 | 3.424 | 0.898 | 3.110 |
| 2.684 | 0.787 | 2.506 | 0.972 | 3.444 | 0.960 | 3.650 | 0.885 | 3.285 |
| 2.955 | 0.775 | 2.640 | 0.970 | 3.642 | 0.961 | 3.874 | 0.881 | 3.481 |
| 3.205 | 0.764 | 2.734 | 0.970 | 3.808 | 0.962 | 4.048 | 0.881 | 3.604 |

**Ln (0.9)**

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| Tau Sq | IVhet MSE | RE MSE | QE MSE | QE-r MSE |  | IVhet var | RE var | QE var | QE-r var |  | IVhet bias2 | RE bias2 | QE bias2 | QE-r bias2 |
| 0.000 | 0.024 | 0.025 | 0.042 | 0.042 |  | 0.023 | 0.024 | 0.042 | 0.042 |  | 0.001 | 0.001 | 0.000 | 0.000 |
| 0.694 | 0.192 | 0.177 | 0.112 | 0.221 |  | 0.180 | 0.173 | 0.110 | 0.217 |  | 0.012 | 0.004 | 0.002 | 0.003 |
| 1.128 | 0.280 | 0.312 | 0.164 | 0.348 |  | 0.260 | 0.306 | 0.160 | 0.342 |  | 0.020 | 0.005 | 0.004 | 0.006 |
| 1.479 | 0.317 | 0.418 | 0.200 | 0.449 |  | 0.291 | 0.414 | 0.194 | 0.441 |  | 0.026 | 0.004 | 0.006 | 0.008 |
| 1.827 | 0.400 | 0.600 | 0.256 | 0.596 |  | 0.369 | 0.594 | 0.249 | 0.587 |  | 0.030 | 0.006 | 0.007 | 0.009 |
| 2.099 | 0.432 | 0.712 | 0.288 | 0.655 |  | 0.393 | 0.705 | 0.278 | 0.642 |  | 0.039 | 0.007 | 0.010 | 0.012 |
| 2.364 | 0.461 | 0.852 | 0.317 | 0.813 |  | 0.424 | 0.848 | 0.307 | 0.802 |  | 0.037 | 0.004 | 0.009 | 0.011 |
| 2.597 | 0.509 | 0.997 | 0.347 | 0.884 |  | 0.470 | 0.993 | 0.335 | 0.871 |  | 0.039 | 0.005 | 0.012 | 0.013 |
| 2.890 | 0.564 | 1.150 | 0.382 | 1.027 |  | 0.522 | 1.146 | 0.370 | 1.016 |  | 0.042 | 0.004 | 0.012 | 0.012 |
| 3.120 | 0.582 | 1.275 | 0.410 | 1.077 |  | 0.534 | 1.270 | 0.396 | 1.060 |  | 0.048 | 0.005 | 0.014 | 0.017 |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Tau sq | RE coverage | RE width | QE coverage | QE width | IVhet coverage | IVhet width | QE-r coverage | QE-r width |
| 0.000 | 0.963 | 0.509 | 0.969 | 0.681 | 0.965 | 0.523 | 0.965 | 0.681 |
| 0.694 | 0.873 | 1.324 | 0.974 | 1.678 | 0.949 | 1.729 | 0.925 | 1.676 |
| 1.128 | 0.849 | 1.618 | 0.973 | 2.088 | 0.954 | 2.201 | 0.914 | 2.067 |
| 1.479 | 0.840 | 1.849 | 0.975 | 2.437 | 0.965 | 2.574 | 0.908 | 2.375 |
| 1.827 | 0.811 | 2.029 | 0.973 | 2.715 | 0.964 | 2.878 | 0.903 | 2.633 |
| 2.099 | 0.805 | 2.193 | 0.972 | 2.960 | 0.969 | 3.147 | 0.902 | 2.869 |
| 2.364 | 0.793 | 2.333 | 0.970 | 3.182 | 0.970 | 3.368 | 0.894 | 3.047 |
| 2.597 | 0.789 | 2.447 | 0.977 | 3.359 | 0.972 | 3.568 | 0.895 | 3.206 |
| 2.890 | 0.776 | 2.595 | 0.973 | 3.589 | 0.972 | 3.807 | 0.888 | 3.416 |
| 3.120 | 0.765 | 2.711 | 0.973 | 3.786 | 0.973 | 3.994 | 0.886 | 3.571 |

**Ln (1.1)**

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| Tau Sq | IVhet MSE | RE MSE | QE MSE | QE-r MSE |  | IVhet var | RE var | QE var | QE-r var |  | IVhet bias2 | RE bias2 | QE bias2 | QE-r bias2 |
| 0.000 | 0.022 | 0.024 | 0.042 | 0.042 |  | 0.022 | 0.023 | 0.042 | 0.042 |  | 0.001 | 0.001 | 0.000 | 0.000 |
| 0.694 | 0.178 | 0.169 | 0.106 | 0.211 |  | 0.172 | 0.167 | 0.105 | 0.208 |  | 0.006 | 0.002 | 0.001 | 0.002 |
| 1.118 | 0.252 | 0.313 | 0.162 | 0.350 |  | 0.244 | 0.311 | 0.160 | 0.347 |  | 0.009 | 0.002 | 0.002 | 0.002 |
| 1.471 | 0.318 | 0.445 | 0.206 | 0.476 |  | 0.303 | 0.442 | 0.203 | 0.471 |  | 0.016 | 0.003 | 0.004 | 0.005 |
| 1.803 | 0.376 | 0.573 | 0.241 | 0.571 |  | 0.356 | 0.568 | 0.236 | 0.564 |  | 0.020 | 0.004 | 0.005 | 0.007 |
| 2.099 | 0.416 | 0.700 | 0.280 | 0.682 |  | 0.399 | 0.697 | 0.276 | 0.677 |  | 0.017 | 0.002 | 0.004 | 0.004 |
| 2.431 | 0.482 | 0.882 | 0.329 | 0.822 |  | 0.463 | 0.880 | 0.323 | 0.817 |  | 0.020 | 0.002 | 0.005 | 0.005 |
| 2.670 | 0.506 | 1.018 | 0.347 | 0.868 |  | 0.484 | 1.014 | 0.341 | 0.862 |  | 0.023 | 0.003 | 0.006 | 0.006 |
| 2.889 | 0.540 | 1.178 | 0.388 | 1.035 |  | 0.514 | 1.173 | 0.380 | 1.025 |  | 0.026 | 0.005 | 0.007 | 0.010 |
| 3.169 | 0.583 | 1.302 | 0.407 | 1.103 |  | 0.558 | 1.299 | 0.399 | 1.094 |  | 0.026 | 0.002 | 0.008 | 0.008 |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Tau sq | RE coverage | RE width | QE coverage | QE width | IVhet coverage | IVhet width | QE-r coverage | QE-r width |
| 0.000 | 0.964 | 0.509 | 0.968 | 0.684 | 0.966 | 0.523 | 0.968 | 0.685 |
| 0.694 | 0.882 | 1.321 | 0.975 | 1.674 | 0.954 | 1.735 | 0.925 | 1.677 |
| 1.118 | 0.849 | 1.621 | 0.975 | 2.099 | 0.962 | 2.207 | 0.915 | 2.075 |
| 1.471 | 0.837 | 1.851 | 0.976 | 2.432 | 0.965 | 2.561 | 0.905 | 2.377 |
| 1.803 | 0.818 | 2.019 | 0.975 | 2.700 | 0.967 | 2.862 | 0.899 | 2.621 |
| 2.099 | 0.808 | 2.186 | 0.976 | 2.950 | 0.971 | 3.133 | 0.903 | 2.847 |
| 2.431 | 0.801 | 2.364 | 0.975 | 3.226 | 0.973 | 3.414 | 0.898 | 3.089 |
| 2.670 | 0.781 | 2.468 | 0.974 | 3.389 | 0.971 | 3.594 | 0.896 | 3.244 |
| 2.889 | 0.770 | 2.607 | 0.973 | 3.613 | 0.976 | 3.824 | 0.890 | 3.439 |
| 3.169 | 0.763 | 2.719 | 0.979 | 3.802 | 0.978 | 4.026 | 0.887 | 3.610 |

**Ln (1.3)**

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| Tau Sq | IVhet MSE | RE MSE | QE MSE | QE-r MSE |  | IVhet var | RE var | QE var | QE-r var |  | IVhet bias2 | RE bias2 | QE bias2 | QE-r bias2 |
| 0.000 | 0.022 | 0.025 | 0.044 | 0.042 |  | 0.022 | 0.024 | 0.044 | 0.042 |  | 0.000 | 0.000 | 0.000 | 0.000 |
| 0.695 | 0.187 | 0.175 | 0.108 | 0.223 |  | 0.183 | 0.173 | 0.108 | 0.221 |  | 0.004 | 0.002 | 0.001 | 0.001 |
| 1.149 | 0.266 | 0.311 | 0.164 | 0.350 |  | 0.261 | 0.309 | 0.163 | 0.348 |  | 0.005 | 0.002 | 0.001 | 0.002 |
| 1.505 | 0.330 | 0.452 | 0.208 | 0.465 |  | 0.321 | 0.451 | 0.206 | 0.463 |  | 0.009 | 0.001 | 0.002 | 0.002 |
| 1.798 | 0.367 | 0.572 | 0.243 | 0.572 |  | 0.356 | 0.568 | 0.240 | 0.568 |  | 0.012 | 0.004 | 0.003 | 0.005 |
| 2.138 | 0.423 | 0.717 | 0.279 | 0.678 |  | 0.414 | 0.715 | 0.278 | 0.676 |  | 0.008 | 0.002 | 0.002 | 0.002 |
| 2.391 | 0.459 | 0.875 | 0.324 | 0.809 |  | 0.449 | 0.874 | 0.322 | 0.807 |  | 0.010 | 0.001 | 0.002 | 0.002 |
| 2.654 | 0.482 | 0.979 | 0.350 | 0.891 |  | 0.471 | 0.977 | 0.348 | 0.887 |  | 0.011 | 0.002 | 0.002 | 0.004 |
| 2.895 | 0.550 | 1.151 | 0.392 | 1.000 |  | 0.535 | 1.149 | 0.388 | 0.996 |  | 0.015 | 0.002 | 0.004 | 0.004 |
| 3.218 | 0.582 | 1.319 | 0.409 | 1.103 |  | 0.568 | 1.315 | 0.405 | 1.096 |  | 0.014 | 0.004 | 0.004 | 0.007 |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Tau sq | RE coverage | RE width | QE coverage | QE width | IVhet coverage | IVhet width | QE-r coverage | QE-r width |
| 0.000 | 0.966 | 0.501 | 0.968 | 0.674 | 0.969 | 0.515 | 0.968 | 0.665 |
| 0.695 | 0.873 | 1.320 | 0.973 | 1.668 | 0.949 | 1.731 | 0.920 | 1.679 |
| 1.149 | 0.857 | 1.637 | 0.974 | 2.123 | 0.957 | 2.235 | 0.910 | 2.091 |
| 1.505 | 0.839 | 1.861 | 0.976 | 2.454 | 0.968 | 2.591 | 0.912 | 2.397 |
| 1.798 | 0.818 | 2.025 | 0.975 | 2.696 | 0.967 | 2.863 | 0.899 | 2.614 |
| 2.138 | 0.809 | 2.193 | 0.977 | 2.955 | 0.972 | 3.140 | 0.905 | 2.859 |
| 2.391 | 0.798 | 2.347 | 0.976 | 3.198 | 0.977 | 3.394 | 0.897 | 3.067 |
| 2.654 | 0.786 | 2.470 | 0.976 | 3.395 | 0.974 | 3.598 | 0.896 | 3.238 |
| 2.895 | 0.785 | 2.603 | 0.976 | 3.598 | 0.976 | 3.825 | 0.894 | 3.428 |
| 3.218 | 0.772 | 2.733 | 0.974 | 3.799 | 0.975 | 4.030 | 0.897 | 3.616 |

**Ln (1.5)**

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Tau Sq | IVhet MSE | RE MSE | QE MSE | QE-r MSE |  | IVhet var | RE var | QE var | QE-r var |  | IVhet bias2 | RE bias2 | QE bias2 | QE-r bias2 |
| 0.000 | 0.022 | 0.024 | 0.043 | 0.042 |  | 0.022 | 0.023 | 0.043 | 0.042 |  | 0.000 | 0.000 | 0.000 | 0.000 |
| 0.717 | 0.185 | 0.176 | 0.116 | 0.223 |  | 0.183 | 0.175 | 0.115 | 0.222 |  | 0.002 | 0.002 | 0.000 | 0.001 |
| 1.154 | 0.264 | 0.313 | 0.162 | 0.361 |  | 0.261 | 0.312 | 0.161 | 0.360 |  | 0.002 | 0.001 | 0.000 | 0.001 |
| 1.498 | 0.309 | 0.436 | 0.205 | 0.456 |  | 0.306 | 0.434 | 0.204 | 0.455 |  | 0.003 | 0.002 | 0.001 | 0.001 |
| 1.840 | 0.383 | 0.592 | 0.252 | 0.604 |  | 0.378 | 0.591 | 0.251 | 0.602 |  | 0.006 | 0.002 | 0.001 | 0.002 |
| 2.139 | 0.406 | 0.703 | 0.282 | 0.684 |  | 0.399 | 0.701 | 0.280 | 0.682 |  | 0.007 | 0.002 | 0.002 | 0.002 |
| 2.416 | 0.461 | 0.855 | 0.318 | 0.794 |  | 0.452 | 0.852 | 0.316 | 0.791 |  | 0.009 | 0.003 | 0.002 | 0.003 |
| 2.675 | 0.500 | 0.982 | 0.354 | 0.879 |  | 0.492 | 0.978 | 0.352 | 0.876 |  | 0.008 | 0.004 | 0.002 | 0.003 |
| 2.902 | 0.537 | 1.156 | 0.391 | 1.012 |  | 0.530 | 1.154 | 0.389 | 1.009 |  | 0.007 | 0.002 | 0.002 | 0.003 |
| 3.229 | 0.576 | 1.310 | 0.413 | 1.120 |  | 0.569 | 1.306 | 0.411 | 1.116 |  | 0.008 | 0.004 | 0.002 | 0.004 |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Tau sq | RE coverage | RE width | QE coverage | QE width | IVhet coverage | IVhet width | QE-r coverage | QE-r width |
| 0.000 | 0.965 | 0.518 | 0.968 | 0.695 | 0.967 | 0.531 | 0.971 | 0.692 |
| 0.717 | 0.878 | 1.345 | 0.973 | 1.701 | 0.952 | 1.754 | 0.926 | 1.697 |
| 1.154 | 0.854 | 1.639 | 0.974 | 2.119 | 0.962 | 2.228 | 0.913 | 2.097 |
| 1.498 | 0.839 | 1.856 | 0.975 | 2.437 | 0.964 | 2.579 | 0.909 | 2.383 |
| 1.840 | 0.821 | 2.047 | 0.972 | 2.723 | 0.969 | 2.895 | 0.901 | 2.656 |
| 2.139 | 0.807 | 2.189 | 0.975 | 2.955 | 0.972 | 3.135 | 0.899 | 2.849 |
| 2.416 | 0.796 | 2.339 | 0.972 | 3.186 | 0.973 | 3.393 | 0.897 | 3.061 |
| 2.675 | 0.792 | 2.498 | 0.975 | 3.419 | 0.975 | 3.629 | 0.902 | 3.281 |
| 2.902 | 0.780 | 2.605 | 0.975 | 3.608 | 0.975 | 3.815 | 0.888 | 3.436 |
| 3.229 | 0.767 | 2.714 | 0.974 | 3.799 | 0.978 | 4.018 | 0.889 | 3.591 |

**Ln (1.7)**

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Tau Sq | IVhet MSE | RE MSE | QE MSE | QE-r MSE |  | IVhet var | RE var | QE var | QE-r var |  | IVhet bias2 | RE bias2 | QE bias2 | QE-r bias2 |
| 0.000 | 0.023 | 0.024 | 0.043 | 0.043 |  | 0.023 | 0.024 | 0.043 | 0.043 |  | 0.000 | 0.000 | 0.000 | 0.000 |
| 0.707 | 0.191 | 0.171 | 0.110 | 0.225 |  | 0.191 | 0.170 | 0.110 | 0.225 |  | 0.000 | 0.001 | 0.000 | 0.000 |
| 1.157 | 0.265 | 0.314 | 0.161 | 0.362 |  | 0.265 | 0.313 | 0.161 | 0.362 |  | 0.001 | 0.002 | 0.000 | 0.001 |
| 1.527 | 0.338 | 0.443 | 0.217 | 0.481 |  | 0.337 | 0.443 | 0.216 | 0.481 |  | 0.001 | 0.001 | 0.000 | 0.000 |
| 1.874 | 0.372 | 0.590 | 0.259 | 0.594 |  | 0.370 | 0.589 | 0.258 | 0.594 |  | 0.002 | 0.001 | 0.000 | 0.000 |
| 2.142 | 0.402 | 0.694 | 0.284 | 0.668 |  | 0.400 | 0.693 | 0.284 | 0.668 |  | 0.002 | 0.001 | 0.000 | 0.001 |
| 2.444 | 0.474 | 0.864 | 0.330 | 0.790 |  | 0.471 | 0.862 | 0.329 | 0.789 |  | 0.003 | 0.002 | 0.001 | 0.001 |
| 2.771 | 0.517 | 1.036 | 0.355 | 0.922 |  | 0.514 | 1.034 | 0.354 | 0.921 |  | 0.003 | 0.002 | 0.001 | 0.001 |
| 3.027 | 0.550 | 1.185 | 0.410 | 0.993 |  | 0.547 | 1.184 | 0.409 | 0.992 |  | 0.003 | 0.001 | 0.001 | 0.001 |
| 3.256 | 0.587 | 1.324 | 0.432 | 1.120 |  | 0.583 | 1.322 | 0.431 | 1.118 |  | 0.004 | 0.002 | 0.001 | 0.001 |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Tau sq | RE coverage | RE width | QE coverage | QE width | IVhet coverage | IVhet width | QE-r coverage | QE-r width |
| 0.000 | 0.970 | 0.514 | 0.969 | 0.689 | 0.972 | 0.528 | 0.969 | 0.692 |
| 0.707 | 0.883 | 1.339 | 0.972 | 1.696 | 0.952 | 1.761 | 0.923 | 1.704 |
| 1.157 | 0.854 | 1.638 | 0.976 | 2.111 | 0.961 | 2.217 | 0.909 | 2.090 |
| 1.527 | 0.840 | 1.873 | 0.976 | 2.464 | 0.964 | 2.608 | 0.907 | 2.409 |
| 1.874 | 0.828 | 2.074 | 0.978 | 2.767 | 0.970 | 2.930 | 0.906 | 2.684 |
| 2.142 | 0.815 | 2.204 | 0.975 | 2.977 | 0.973 | 3.158 | 0.904 | 2.874 |
| 2.444 | 0.796 | 2.368 | 0.975 | 3.223 | 0.975 | 3.435 | 0.904 | 3.106 |
| 2.771 | 0.793 | 2.528 | 0.977 | 3.460 | 0.977 | 3.668 | 0.898 | 3.317 |
| 3.027 | 0.787 | 2.645 | 0.974 | 3.651 | 0.978 | 3.872 | 0.894 | 3.494 |
| 3.256 | 0.771 | 2.749 | 0.975 | 3.818 | 0.978 | 4.069 | 0.891 | 3.621 |

**Ln (1.9)**

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Tau Sq | IVhet MSE | RE MSE | QE MSE | QE-r MSE |  | IVhet var | RE var | QE var | QE-r var |  | IVhet bias2 | RE bias2 | QE bias2 | QE-r bias2 |
| 0.000 | 0.023 | 0.025 | 0.044 | 0.044 |  | 0.023 | 0.025 | 0.044 | 0.044 |  | 0.000 | 0.000 | 0.000 | 0.000 |
| 0.719 | 0.191 | 0.171 | 0.114 | 0.227 |  | 0.191 | 0.170 | 0.114 | 0.227 |  | 0.000 | 0.001 | 0.000 | 0.000 |
| 1.177 | 0.256 | 0.299 | 0.166 | 0.347 |  | 0.256 | 0.299 | 0.166 | 0.347 |  | 0.000 | 0.000 | 0.000 | 0.000 |
| 1.556 | 0.344 | 0.453 | 0.222 | 0.474 |  | 0.343 | 0.452 | 0.222 | 0.474 |  | 0.000 | 0.001 | 0.000 | 0.000 |
| 1.880 | 0.367 | 0.602 | 0.255 | 0.571 |  | 0.367 | 0.601 | 0.255 | 0.571 |  | 0.000 | 0.001 | 0.000 | 0.000 |
| 2.201 | 0.423 | 0.723 | 0.289 | 0.693 |  | 0.423 | 0.722 | 0.289 | 0.693 |  | 0.000 | 0.000 | 0.000 | 0.000 |
| 2.530 | 0.460 | 0.865 | 0.304 | 0.797 |  | 0.460 | 0.864 | 0.304 | 0.797 |  | 0.000 | 0.001 | 0.000 | 0.000 |
| 2.750 | 0.523 | 1.035 | 0.361 | 0.932 |  | 0.522 | 1.035 | 0.361 | 0.932 |  | 0.000 | 0.001 | 0.000 | 0.000 |
| 2.959 | 0.558 | 1.136 | 0.388 | 1.016 |  | 0.557 | 1.135 | 0.388 | 1.016 |  | 0.001 | 0.001 | 0.000 | 0.000 |
| 3.340 | 0.614 | 1.363 | 0.437 | 1.160 |  | 0.613 | 1.362 | 0.437 | 1.160 |  | 0.001 | 0.000 | 0.000 | 0.000 |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Tau sq | RE coverage | RE width | QE coverage | QE width | IVhet coverage | IVhet width | QE-r coverage | QE-r width |
| 0.000 | 0.967 | 0.520 | 0.969 | 0.695 | 0.970 | 0.534 | 0.967 | 0.696 |
| 0.719 | 0.887 | 1.350 | 0.974 | 1.704 | 0.954 | 1.761 | 0.924 | 1.709 |
| 1.177 | 0.861 | 1.651 | 0.972 | 2.136 | 0.960 | 2.250 | 0.914 | 2.113 |
| 1.556 | 0.844 | 1.887 | 0.972 | 2.483 | 0.965 | 2.638 | 0.913 | 2.435 |
| 1.880 | 0.815 | 2.048 | 0.973 | 2.729 | 0.967 | 2.890 | 0.899 | 2.648 |
| 2.201 | 0.810 | 2.219 | 0.974 | 2.999 | 0.971 | 3.188 | 0.903 | 2.892 |
| 2.530 | 0.806 | 2.366 | 0.977 | 3.223 | 0.973 | 3.428 | 0.898 | 3.088 |
| 2.750 | 0.791 | 2.529 | 0.971 | 3.465 | 0.972 | 3.670 | 0.894 | 3.313 |
| 2.959 | 0.781 | 2.609 | 0.975 | 3.614 | 0.977 | 3.829 | 0.897 | 3.426 |
| 3.340 | 0.780 | 2.792 | 0.976 | 3.887 | 0.980 | 4.123 | 0.900 | 3.699 |

**Ln (2.1)**

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Tau Sq | IVhet MSE | RE MSE | QE MSE | QE-r MSE |  | IVhet var | RE var | QE var | QE-r var |  | IVhet bias2 | RE bias2 | QE bias2 | QE-r bias2 |
| 0.000 | 0.024 | 0.026 | 0.046 | 0.045 |  | 0.024 | 0.026 | 0.046 | 0.045 |  | 0.000 | 0.000 | 0.000 | 0.000 |
| 0.731 | 0.200 | 0.179 | 0.117 | 0.235 |  | 0.200 | 0.178 | 0.117 | 0.235 |  | 0.000 | 0.000 | 0.000 | 0.000 |
| 1.187 | 0.262 | 0.305 | 0.164 | 0.353 |  | 0.262 | 0.304 | 0.164 | 0.353 |  | 0.000 | 0.001 | 0.000 | 0.000 |
| 1.567 | 0.338 | 0.454 | 0.219 | 0.482 |  | 0.338 | 0.453 | 0.218 | 0.482 |  | 0.000 | 0.001 | 0.000 | 0.000 |
| 1.905 | 0.386 | 0.595 | 0.246 | 0.612 |  | 0.386 | 0.595 | 0.246 | 0.612 |  | 0.000 | 0.000 | 0.000 | 0.000 |
| 2.184 | 0.426 | 0.722 | 0.295 | 0.680 |  | 0.426 | 0.720 | 0.295 | 0.680 |  | 0.000 | 0.002 | 0.000 | 0.000 |
| 2.517 | 0.457 | 0.877 | 0.319 | 0.800 |  | 0.457 | 0.876 | 0.319 | 0.800 |  | 0.000 | 0.001 | 0.000 | 0.000 |
| 2.761 | 0.512 | 0.990 | 0.352 | 0.902 |  | 0.511 | 0.990 | 0.352 | 0.902 |  | 0.000 | 0.000 | 0.000 | 0.000 |
| 3.019 | 0.561 | 1.190 | 0.391 | 1.023 |  | 0.561 | 1.189 | 0.391 | 1.023 |  | 0.000 | 0.001 | 0.000 | 0.000 |
| 3.381 | 0.589 | 1.276 | 0.409 | 1.111 |  | 0.589 | 1.275 | 0.409 | 1.111 |  | 0.000 | 0.001 | 0.000 | 0.000 |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Tau sq | RE coverage | RE width | QE coverage | QE width | IVhet coverage | IVhet width | QE-r coverage | QE-r width |
| 0.000 | 0.965 | 0.530 | 0.968 | 0.705 | 0.966 | 0.545 | 0.967 | 0.706 |
| 0.731 | 0.885 | 1.366 | 0.973 | 1.730 | 0.952 | 1.800 | 0.924 | 1.732 |
| 1.187 | 0.862 | 1.647 | 0.973 | 2.123 | 0.961 | 2.245 | 0.914 | 2.111 |
| 1.567 | 0.837 | 1.883 | 0.973 | 2.479 | 0.962 | 2.619 | 0.910 | 2.421 |
| 1.905 | 0.824 | 2.074 | 0.975 | 2.758 | 0.968 | 2.926 | 0.905 | 2.685 |
| 2.184 | 0.812 | 2.225 | 0.974 | 2.993 | 0.973 | 3.178 | 0.905 | 2.893 |
| 2.517 | 0.803 | 2.371 | 0.978 | 3.230 | 0.975 | 3.433 | 0.899 | 3.091 |
| 2.761 | 0.796 | 2.519 | 0.978 | 3.458 | 0.975 | 3.670 | 0.898 | 3.300 |
| 3.019 | 0.780 | 2.643 | 0.977 | 3.657 | 0.977 | 3.885 | 0.896 | 3.486 |
| 3.381 | 0.783 | 2.776 | 0.976 | 3.854 | 0.978 | 4.090 | 0.900 | 3.675 |

**Ln (2.3)**

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Tau Sq | IVhet MSE | RE MSE | QE MSE | QE-r MSE |  | IVhet var | RE var | QE var | QE-r var |  | IVhet bias2 | RE bias2 | QE bias2 | QE-r bias2 |
| 0.000 | 0.024 | 0.026 | 0.045 | 0.044 |  | 0.024 | 0.026 | 0.045 | 0.044 |  | 0.000 | 0.000 | 0.000 | 0.000 |
| 0.731 | 0.194 | 0.178 | 0.115 | 0.232 |  | 0.193 | 0.178 | 0.115 | 0.232 |  | 0.001 | 0.000 | 0.000 | 0.000 |
| 1.216 | 0.287 | 0.325 | 0.166 | 0.369 |  | 0.287 | 0.324 | 0.166 | 0.369 |  | 0.000 | 0.001 | 0.000 | 0.000 |
| 1.565 | 0.340 | 0.450 | 0.215 | 0.494 |  | 0.340 | 0.449 | 0.215 | 0.494 |  | 0.000 | 0.001 | 0.000 | 0.000 |
| 1.901 | 0.381 | 0.565 | 0.259 | 0.564 |  | 0.380 | 0.564 | 0.259 | 0.564 |  | 0.000 | 0.000 | 0.000 | 0.000 |
| 2.247 | 0.436 | 0.706 | 0.297 | 0.697 |  | 0.435 | 0.706 | 0.297 | 0.696 |  | 0.001 | 0.000 | 0.000 | 0.001 |
| 2.534 | 0.483 | 0.888 | 0.333 | 0.812 |  | 0.482 | 0.888 | 0.332 | 0.811 |  | 0.001 | 0.000 | 0.001 | 0.001 |
| 2.863 | 0.525 | 0.994 | 0.355 | 0.884 |  | 0.524 | 0.994 | 0.354 | 0.883 |  | 0.001 | 0.000 | 0.001 | 0.000 |
| 3.066 | 0.563 | 1.175 | 0.390 | 1.001 |  | 0.562 | 1.175 | 0.390 | 1.000 |  | 0.001 | 0.000 | 0.000 | 0.001 |
| 3.338 | 0.598 | 1.248 | 0.423 | 1.063 |  | 0.597 | 1.248 | 0.423 | 1.062 |  | 0.001 | 0.000 | 0.000 | 0.001 |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Tau sq | RE coverage | RE width | QE coverage | QE width | IVhet coverage | IVhet width | QE-r coverage | QE-r width |
| 0.000 | 0.967 | 0.529 | 0.971 | 0.705 | 0.968 | 0.544 | 0.971 | 0.704 |
| 0.731 | 0.888 | 1.367 | 0.971 | 1.727 | 0.949 | 1.791 | 0.924 | 1.731 |
| 1.216 | 0.860 | 1.678 | 0.976 | 2.168 | 0.960 | 2.287 | 0.914 | 2.143 |
| 1.565 | 0.847 | 1.893 | 0.975 | 2.479 | 0.965 | 2.633 | 0.914 | 2.439 |
| 1.901 | 0.827 | 2.050 | 0.973 | 2.720 | 0.967 | 2.896 | 0.907 | 2.654 |
| 2.247 | 0.820 | 2.232 | 0.973 | 3.002 | 0.969 | 3.197 | 0.905 | 2.914 |
| 2.534 | 0.798 | 2.387 | 0.975 | 3.237 | 0.972 | 3.454 | 0.893 | 3.121 |
| 2.863 | 0.801 | 2.549 | 0.978 | 3.493 | 0.976 | 3.718 | 0.906 | 3.353 |
| 3.066 | 0.780 | 2.659 | 0.974 | 3.679 | 0.974 | 3.906 | 0.898 | 3.506 |
| 3.338 | 0.782 | 2.756 | 0.976 | 3.815 | 0.977 | 4.070 | 0.897 | 3.640 |

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