

Appendix: Mathematical Derivations of the Error with Lateral Pin and Score-Mark Techniques

Our primary recommendation is to create an osteotomy in the plane of the femoral neck or use a pin in the femoral neck to judge anteversion. For the purposes of this study and for surgeons who do not wish to follow this recommendation, we have provided descriptions of the mathematical processes for calculating error with lateral pin and score-mark techniques. As these calculations are fairly complex for routine use, we have developed an online calculator to simplify their clinical application, which is available at <http://www.femoralversion.com>.

Calculation of the Error from a Change in Femoral Version

To calculate the inadvertent change in femoral version from a coronal plane osteotomy, it is important to realize that femoral neck inclination (the true anatomic angle between the femoral neck and the femoral shaft) does not change with a coronal plane osteotomy (Fig. 5)³.

Prior to the osteotomy, apparent neck-shaft angle (ANSA) and preoperative version are measured. Inclination is then calculated with use of a previously derived relationship between inclination, version, and ANSA³:

$$(1) \text{ Inclination} = \tan^{-1}(\tan(\text{version}) \cdot \cos(\text{ANSA} - 90))$$

After the osteotomy, ANSA decreases by the amount of the varus osteotomy while inclination remains constant. Thus, postoperative version can then be calculated by manipulating the above formula into the one below:

$$(2) \text{ Postoperative version} = \tan^{-1}(\tan(\text{inclination}) / \cos(\text{postoperative ANSA} - 90))$$

The inadvertent change in version is the difference between preoperative and postoperative version. This error occurs in the lateral pin and score-mark techniques but not in the neck-pin technique since the neck pin itself changes with femoral version.

Calculation of the Additional Error from Use of Lateral Pins

After the coronal plane osteotomy, the proximal lateral pin is brought out of the plane of the distal lateral pin. This mal-oriens the pin with respect to the condylar axis in the axial plane. To determine the resulting error, the following calculations need to be performed.

The angle of the proximal pin with respect to the shaft is initially 90° in the coronal plane. If one pretends that the pin itself is a femoral neck, both the “inclination” and “version” of the pin are equal to the preoperative version of the femur, since the pin is parallel to the neck pin the axial plane and since inclination and version are equal when ANSA is 90° (based on formula 1 above with ANSA set at 90°).

After a coronal plane osteotomy, the “ANSA” of the pin is equal to 90° minus the magnitude of the varus osteotomy. In this case, “inclination” remains constant (Fig. 4), and the “version” of the pin increases based on formula 2. The difference between preoperative and postoperative “version” of the pin can now be calculated and added to the error from the change in femoral version to calculate the total error of the lateral pin technique.

$$(3) \text{ Postoperative pin version} = \tan^{-1}(\tan(\text{inclination}) / \cos((90 - \text{osteotomy angle}) - 90)) \text{ or}$$

$$(4) \text{ Postoperative pin version} = \tan^{-1}(\tan(\text{inclination}) / \cos(-\text{osteotomy angle}))$$

In contrast, after a neck plane osteotomy, the “TNSA” of the pin changes while version does not. Thus, the pin is not brought out of plane and no error is derived.

It is important to note that the additional error from the lateral pin technique is based on placing a lateral pin in the plane of the femoral neck and out of plane from the osteotomy. If a surgeon places the lateral pin in the same plane as the osteotomy, such as with the neck plane osteotomy in this study, or directly lateral in the coronal plane for a coronal plane osteotomy, then this additional error will not occur; however, any error from an inadvertent change in femoral version (see above) will still occur.

Calculation of the Additional Error with Anterior Pins Representing a Score Mark

When seeing the proximal anterior score-mark pin in the anteroposterior view, the pin is pointing anteriorly toward the viewer. To simplify the analysis, the pin is first theoretically rotated 90° on the axis of the femoral shaft so that it is pointing from lateral to medial. It can then be modeled as a femur with an ANSA of 90°. The starting “version” of the pin is more complicated. If one defines the “bicondylar axis” by the angle of the osteotomy, then “version” is defined as the angle between the medial-facing pin and the osteotomy orientation in the axial plane. Since the “bicondylar axis” is defined by the plane of the osteotomy, both neck plane and coronal plane osteotomies can be modeled similarly, and one can then approach this problem using the exact same mathematical steps as with the lateral pin above. At the conclusion, the proximal anterior pin can be theoretically rotated 90° back into its more anterior orientation, with the change in version calculated still reflecting the amount of error from bringing the pin out of plane. It is important to note that when the pin enters the bone more anteromedial than the osteotomy plane it is subtractive from the error derived from change in version, and when it is more anterolateral than the osteotomy plane it is additive to the error derived from change in version. ■



Fig. E-1

Comparison of neck plane and coronal plane osteotomies. **Fig. E1-A** In a neck plane osteotomy, the femur is rotated so that the femoral neck is parallel to the table. Note that the lateral femoral condyle is elevated off the table as anteversion is neutralized. The osteotomy is then performed from directly lateral, with the wedge in the plane of the neck. **Fig. E1-B** In a coronal plane osteotomy, the femur rests with the posterior aspects of the femoral condyles flat on the table, theoretically with the patella pointing toward the ceiling. Note that the femoral neck elevates out of plane because of femoral anteversion. The osteotomy is performed from lateral and is not in the plane of the femoral neck. Thus, femoral version can be altered without any purposeful rotation, similar to the effect seen in Figure 1.

TABLE E-1 Measurements of Neck-Shaft Angle and Version in the Sawbone Femora*

Specimen No.	Osteotomy Plane	Preop. ANSA (deg)	Preop. Version (deg)	Postop. ANSA (deg)	Postop. Version (deg)	Change in Version† (deg)	Change in Alignment (Theoretical Value)†‡ (deg)		
							Neck Pin	Score Mark	Lateral Pin
Low valgus									
1	Coronal	135	49	111	4	45	48 (45)	40 (39)	39 (34)
2	Coronal	132	41	112	−7	48	45 (48)	41 (44)	37 (40)
3	Coronal	128	32	101	−2	34	29 (34)	31 (31)	28 (25)
4	Coronal	128	33	108	1	32	30 (32)	27 (29)	25 (25)
5	Coronal	126	22	100	0	22	23 (22)	26 (20)	21 (16)
6	Coronal	124	22	102	1	21	16 (21)	21 (20)	19 (16)
7	Neck	134	47	96	−2	49	47 (49)	47 (48)	48 (49)
8	Neck	130	37	100	−4	41	41 (41)	41 (41)	39 (41)
9	Neck	133	42	100	−3	45	45 (45)	45 (44)	39 (45)
10	Neck	128	32	106	−5	37	32 (37)	35 (37)	36 (37)
11	Neck	125	19	101	2	17	16 (17)	20 (18)	17 (17)
12	Neck	127	24	103	−3	27	25 (27)	29 (27)	28 (27)
High valgus									
13	Coronal	160	45	108	−12	57	53 (57)	45 (45)	15 (19)
14	Coronal	153	40	110	−3	43	41 (43)	26 (26)	15 (16)
15	Coronal	143	27	98	−9	36	37 (36)	30 (29)	24 (17)
16	Coronal	141	30	100	−2	32	28 (32)	15 (25)	10 (15)
17	Coronal	154	50	107	−4	54	56 (54)	37 (35)	24 (21)
18	Coronal	156	46	116	−2	48	43 (48)	23 (28)	15 (17)
19	Neck	159	42	113	−7	49	51 (49)	37 (39)	37 (49)
20	Neck	151	37	108	−8	45	48 (45)	43 (39)	39 (45)
21	Neck	144	34	104	−4	38	35 (38)	22 (30)	29 (38)
22	Neck	149	33	101	−8	41	40 (41)	34 (34)	28 (41)
23	Neck	150	46	103	−9	55	49 (55)	41 (47)	45 (55)
24	Neck	151	51	111	−3	54	51 (54)	41 (47)	46 (54)

*ANSA = apparent neck-shaft angle. †The error for each measurement technique can be calculated by the difference between change in version and the change in pin alignment. ‡The values in parentheses represent the theoretical values for each cell based on the mathematical model.