APPENDIX

Calculating volume scotomas for patients with central scotomas

This appendix provides formulas to derive the shape and extent of volume scotomas for some simple examples of bilateral central field loss. The main purpose of these calculations is not to provide a method for calculating the scotoma for a particular patient, but rather to derive quantitative measures for typical volume scotomas, as this is needed to provide a general appreciation of their potential impact on function.

For all the cases discussed below, we assume that the monocular scotomas are equal in diameter, centered on the position of the fovea, and the PRL is centered just below the scotoma in both eyes. While calculations for more complex cases are possible, they are not likely to be a practical substitute for direct measurement when evaluating and advising individual patients. Binocular volume scotomas can also exist at overlaps of the CFL scotomas and the physiological blind spot (ONH) scotomas, although these are primarily associated with extremely close fixation distance or very large central scotomas. Formulas for these volume scotomas are also provided.

In all cases illustrated below, θ represents the convergence angle, and ψ represents the horizontal angular size of the monocular scotomas. f represents the distance from the eyes to the point of fixation, a represents the distance from the eyes to the proximal limit of the anterior scotoma, and p represents the distance to the end of the posterior scotoma (if finite). For the simple scotomas illustrated, their geometry is completely determined by θ, ψ, and the interpupillary distance (IPD). When shown, the ONH scotomas assume an offset of 15° from fixation, an ONH width of 5°, and an IPD of 6.6cm.

Case I: Convergence angle = 0º (fixation at infinity)

When a patient fixates at a great distance, the visual axes of the eyes are essentially parallel and the convergence angle is essentially zero. In this case, only the inner edges of the monocular scotomas will overlap and form the anterior volume scotoma (Fig. 3a and A1). The anterior volume scotoma extends to infinity.

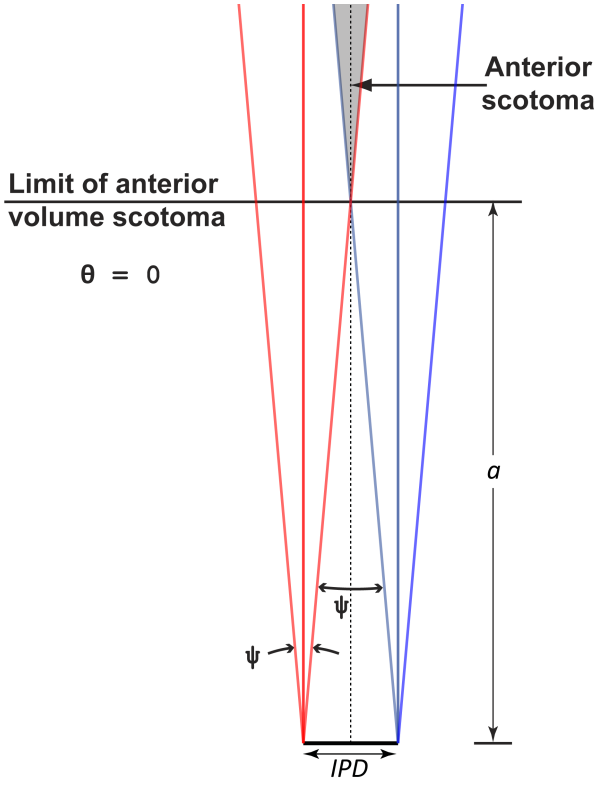

Figure A1: Fixating at infinity. Only an anterior volume scotoma is present and it extends to infinity.

With $\theta = 0$, there is only an anterior scotoma. It spreads to infinity with angle ψ, starting at a distance from the eyes given as:

$$a = \frac{IPD}{2 \times tan\left(\frac{\psi}{2}\right)} \ . \tag{A1}$$

Note that the smaller the central scotoma size, the farther the distance to the start of the anterior scotoma, and hence the apparent angular width (not) seen by the eyes can be much smaller than ψ at intermediate distances.

The ONH scotomas do not intersect the fellow eye's CFL scotoma unless the CFL scotomas are larger than 25 degrees, at which point the ipsilateral CFL scotomas are starting to overlap their eye's ONH, so no additional bilateral loss is contributed by the ONHs.

Case II: Convergence angle < central scotoma angle

When the convergence angle is smaller than the central scotoma angle, there is a finite anterior volume scotoma and infinite diverging posterior volume scotoma (Fig. A2).

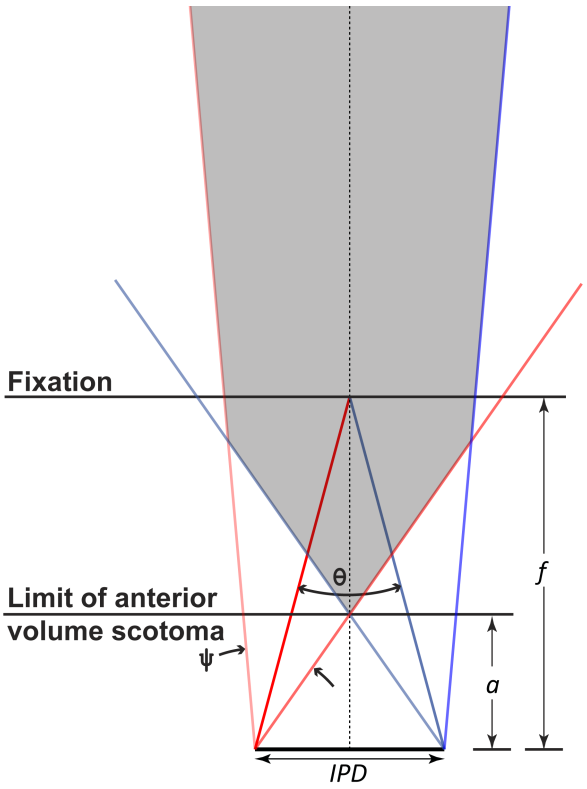

Figure A2: When the convergence angle is smaller than the binocular scotoma angular size, there is a limited anterior scotoma and infinite posterior scotoma.

The anterior scotoma diverges with angle $\theta+\psi$, while the posterior scotoma diverges at a narrower angle, ψ-θ. The distance from the eyes to the start of the anterior scotoma is given by:

$$a = \frac{IPD}{2\times tan\left(\frac{\theta+\psi}{2}\right)} \quad . \tag{A2}$$

The transition from the diverging anterior scotoma to the posterior scotoma width actually starts slightly anterior to the fixation plane. That distance differs from the fixation distance f by a factor of $1 - \tan^2(\psi)$.

For this case, the ONH and CFL scotomas do not intersect if the CFL scotomas are less than 8.33° (solving Eq. A4 for s_c=∞), and if greater, generally extend to infinity.

Case III: Convergence angle = central scotoma size

When fixating so that the convergence angle equals the angular size of the scotoma, then the outer edges of the monocular scotomas are parallel. The volume scotoma forms a "tunnel scotoma" behind the fixated plane that remains fixed in linear width. The tunnel scotoma extends to infinity. It has the same linear width as the IPD (Fig. 3b and A3).

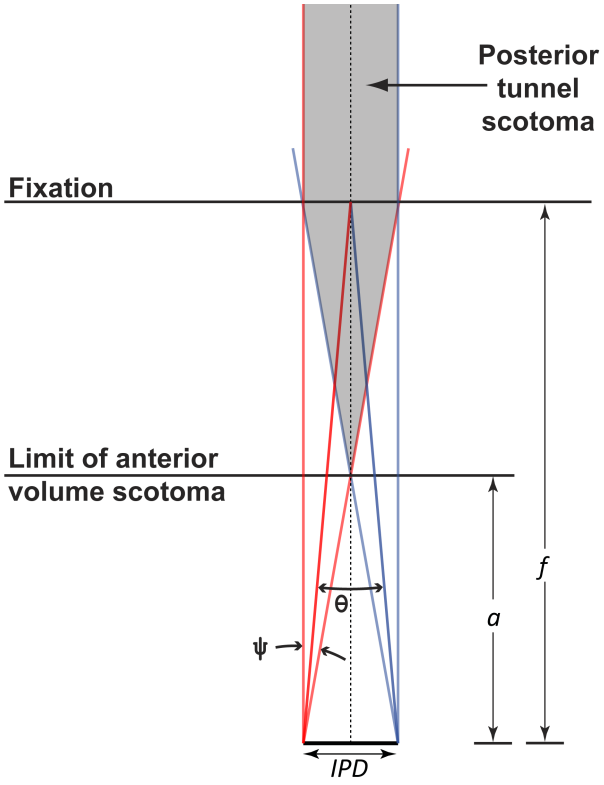

Figure A3: When the convergence angle equals the scotoma angular size, a tunnel scotoma of constant linear width forms.

An anterior scotoma diverging with angle θ starts at a distance from the eyes given by Eq. A2. As above, the transition occurs anterior to fixation.

When the convergence angle and therefore, in this case, the central scotoma size, is greater than 8.33° (a fixation distance of 45 cm or less for a patient with of 6.6cm IPD), the ONH and CFL scotomas intersect, forming binocular scotomas that extend to infinity (main text Fig. 3b).

Case IV: Convergence angle > central scotoma size

When fixating with a convergence angle greater than the angular size of the central scotoma (Fig. 3c and A4), a zone of volume scotoma is formed from the overlap of the inner and outer edges of the monocular scotomas.

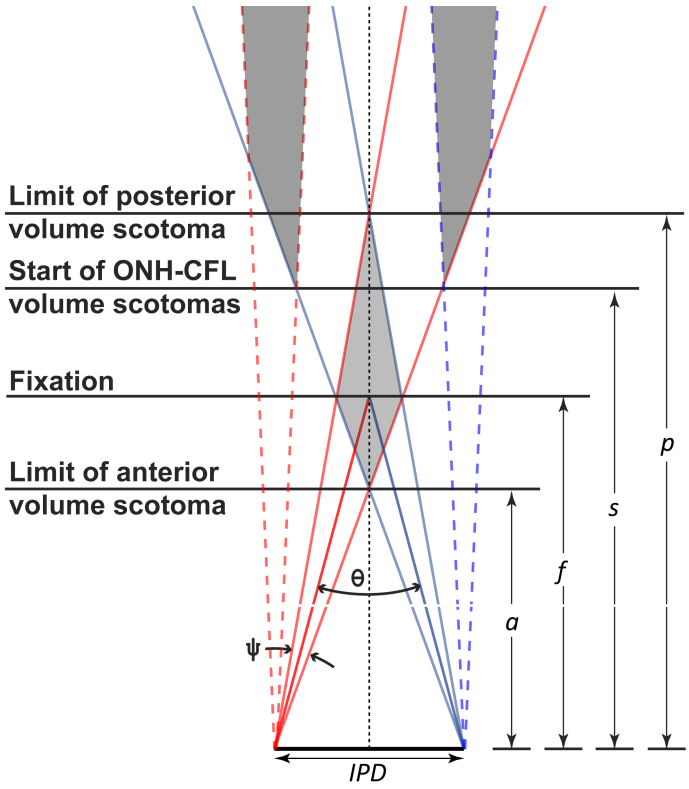

Figure A4: When the convergence angle is greater than the binocular scotoma angular size, there are finite anterior and posterior volume scotomas. There can also be ONH-CFL scotoma overlaps. For the illustrated configuration, the ONH-CFL overlapping areas are finite but extend well beyond the figure boundary.

The anterior volume scotoma diverges with angle $\theta + \psi$. The posterior volume scotoma converges with angle θ - ψ. As above, the transition occurs slightly anterior to fixation. The anterior scotoma starts at distance a from the eyes given by Eq. A2, while the posterior scotoma ends at distance p from the eyes given by Eq. A3.

$$p = \frac{IPD}{2\times tan\left(\frac{\theta-\psi}{2}\right)} \quad . \tag{A3}$$

The ONH and CFL scotomas can intersect if the convergence angle is greater than 8.33° (a fixation distance of 45cm for an IPD of 6.6cm) and the scotoma width is only slightly less than the convergence angle . The overlap extends to infinity for essentially all scotoma sizes.

Calculating the ONH/CFL scotoma intersections

Overlaps between an ONH scotoma and the contralateral CFL scotoma to produce a binocular volume scotoma are bounded by the sight lines of the ONH and CFL scotoma edges, which can intersect at four points. If all four points are at finite distances, the enclosed area forms a skewed kite-shaped scotoma

The distance to the closest intersection, s_c, is given in Eq. A4, if the solution is positive.

$$s_c = \frac{IPD}{tan\left(\frac{\theta}{2}+\frac{\psi}{2}\right) - tan\left(\frac{\theta}{2}-\left(O_o-\frac{O_w}{2}\right)\right)} \ , \tag{A4}$$

where *IPD*, θ and ψ are as above, O_o is the angular distance of the ONH center from fixation (usually 15°), and O_w is the ONH width (~5°). All are in degrees.

If the closest intersection does not exist ($s_c < 0$) there is no volume scotoma that includes the ONH. The distance to the farthest intersection, s_f, is given in Eq. A5. If the farthest intersection does not exist at a finite distance in front of the eyes ($s_f < 0$), the scotoma extends to infinity.

$$s_f = \frac{IPD}{tan\left(\frac{\theta}{2}-\frac{\psi}{2}\right) - tan\left(\frac{\theta}{2}-\left(O_o+\frac{O_w}{2}\right)\right)} \ , \tag{A5}$$

The temporal limit to the scotoma width, s_t, is given in Eq. A6, and the distance to the nasal limit of the width, s_n, is given in Eq. A7. The intersections exist if the corresponding equation is positive.

$$s_t = \frac{IPD}{tan\left(\frac{\theta}{2}+\frac{\psi}{2}\right) - tan\left(\frac{\theta}{2}-\left(O_o+\frac{O_w}{2}\right)\right)} \tag{A6}$$

$$s_n = \frac{IPD}{tan\left(\frac{\theta}{2}-\frac{\psi}{2}\right) - tan\left(\frac{\theta}{2}-\left(O_o-\frac{O_w}{2}\right)\right)} \tag{A7}$$

Figure A5 shows the convergence angles and CFL scotoma widths for which each of these intersection points exists, assuming an IPD of 6.6cm, O_o= 15°, and O_w= 5°. An intersection exists at a finite distance for all points above its corresponding line.

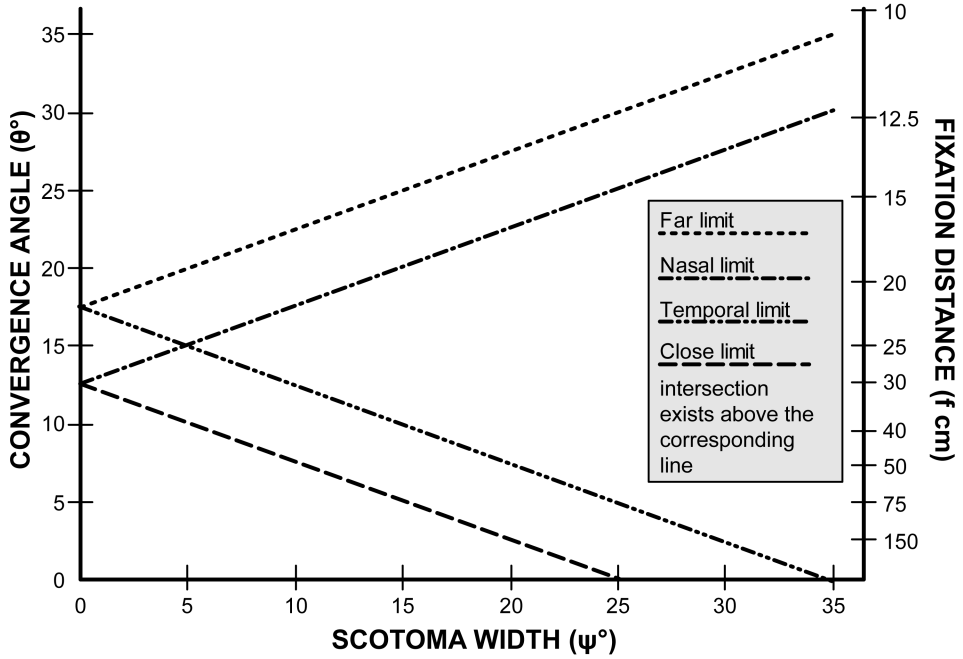

Figure A5: Convergence angles (or corresponding fixation distances) and scotoma widths at which the ONH scotoma intersects the contralateral CFL scotoma. Lines for the close, far, temporal, and nasal intersection points are shown. An intersection exists at a finite distance for all points above its corresponding line. ONHs are assumed to be centered 15° temporal of fixation, with a width of 5° and an IPD of 6.6cm. The close and temporal limit lines go negative at the point that the ipsilateral CFL scotoma begins to and then fully overlaps the ONH scotoma, making the ONH interaction moot.