APPENDIX to Harris WF & Evans T Chromatic Aberration in Heterocentric Astigmatic Systems Including the Eye Optom Vis Sci 2012;89

The theory is illustrated here for a model eye with four tilted astigmatic refracting surfaces. The optical system is the visual optical system of the eye from immediately in front of the cornea to immediately in front of the retina. The curvatures, tilts, and separations are listed in Table A1. K1 and K2 are the first and second surfaces of the cornea and L1 and L2 are the first and second surfaces of the lens of the eye. K1 has principal meridians at $180°$ and $90°$; the radii of curvature along them are 6.5 and 8 mm respectively. The horizontal and vertical components of tilt of K1 are 0.06 and -0.05 (radians) respectively; the right side of the cornea would be tilted away from and the top towards an observer looking at the eye. We use the equations for refractive index of the cornea, aqueous, lens, and vitreous published by Villegas, Carretero, and Fimia[1]. The index in front of the eye is $n_0 = 1$. We use the vacuum wavelengths 656.3 nm for red and 486.1 nm for blue respectively that have been used by others[2]. In order to show small differences we retain more digits than may be physically meaningful.

TABLE A1.

Principal radii of curvature, separation, and tilt of surfaces of the model eye used in the numerical example.

Surface	Principal radii mm{degr}mm	Separation mm	Tilt
K1	$6.5\{180\}8$		$(0.06 \quad -0.05)^{\mathrm{T}}$
		0.5	
K2	$5.8\{20\}7.2$		$(0.04 \quad 0.06)^{\mathrm{T}}$
		3	
L1	$10.2\{100\}8.7$		$(-0.07 \quad 0.1)^{\mathrm{T}}$
		4	
L2	$-4.5\{70\}-6.5$		$(-0.05 \quad -0.03)^{\mathrm{T}}$
		16.5	

The transferences calculated by the method described elsewhere[3] for red and blue light turn out to be

$$\mathbf{T}_\mathrm{r} = \begin{pmatrix} -0.1513 & -0.0134 & 16.5690 & -0.1292 & -0.3134 \\ -0.0121 & -0.0125 & -0.1290 & 16.3221 & 0.1936 \\ -0.0686 & -0.0012 & 0.9046 & -0.0104 & -0.0176 \\ -0.0011 & -0.0619 & -0.0103 & 0.8840 & 0.0084 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{A1}$$

and

$$\mathbf{T}_\mathrm{b} = \begin{pmatrix} -0.1687 & -0.0136 & 16.4641 & -0.1309 & -0.3185 \\ -0.0123 & -0.0279 & -0.1307 & 16.2137 & 0.1957 \\ -0.0700 & -0.0012 & 0.9030 & -0.0106 & -0.0179 \\ -0.0011 & -0.0632 & -0.0105 & 0.8820 & 0.0086 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{A2}$$

Entries in the last three columns of the first two rows are in millimetres; entries in the first two columns of the third and fourth rows are in kilodiopters.

Distant Object Point

Consider a distant object point O in a vertical plane containing longitudinal axis Z. Rays from O arrive at the model eye with inclination $\mathbf{a}_\mathrm{O} = \begin{pmatrix} 0 \\ -0.05 \end{pmatrix}$ relative to Z. Details of the calculation are summarized in Table A2. For example, the red blurred image has a near vertical line (it is at about $94.90°$) 2.9567 mm in front of the retina; the other line is 0.2455 mm in front of the retina. (The blue image has a line at $94.87°$, not quite the same as for the red image.) The longitudinal chromatic aberration is $\Delta\mathbf{Z}$ as listed. Its principal structure is -0.3235 mm along $96.96°$ and -0.2884 mm. The horizontal and vertical components of the transverse chromatic aberration are -0.0008 mm and 0.0160 mm respectively.

It may also be of interest to calculate the chromatic difference of refractive compensation for the eye. The refractive compensation is given by[4] $\mathbf{F}_0 = \mathbf{B}^{-1}\mathbf{A}$, a dioptric power matrix. We obtain it from the transferences (Eqs. A1 and A2). In conventional spherocylindrical terms the results are $-0.6958 \;\; -8.5194 \times 95.50$ and $-1.6481 \;\; -8.6850 \times 95.59$ for red and blue light respectively. The chromatic difference of refractive compensation turns out to be

TABLE A2.

Longitudinal $\Delta\mathbf{Z}$ and transverse $\Delta\mathbf{y}$ chromatic aberration of a model heterocentric astigmatic eye and a distant object point with $\mathbf{a}_O = \begin{pmatrix} 0 \\ -0.05 \end{pmatrix}$.

	red	blue
$\mathbf{L}$ (kD) (Eq.11)	$\begin{pmatrix} -0.4874 & 0.4236 \\ 0.4236 & -5.3960 \end{pmatrix}$	$\begin{pmatrix} -0.4270 & 0.1643 \\ 0.1643 & -2.3414 \end{pmatrix}$
$\mathbf{Z}$ (mm) (Eq.4)	$\begin{pmatrix} -2.9369 & -0.2306 \\ -0.2306 & -0.2653 \end{pmatrix}$	$\begin{pmatrix} -3.2258 & -0.2263 \\ -0.2263 & -0.5882 \end{pmatrix}$
z_- (mm)	-2.9567	-3.2451
z_+ (mm)	-0.2455	-0.5960
$\mathbf{v}_+$	$\begin{pmatrix} 0.0853 \\ -0.9964 \end{pmatrix}$	$\begin{pmatrix} 0.0849 \\ -0.9964 \end{pmatrix}$
$\mathbf{V}$ (mm) (Eq. 26)	$\begin{pmatrix} -2.2021 & -0.1729 \\ -0.1729 & -0.1989 \end{pmatrix}$	$\begin{pmatrix} -2.4070 & -0.1689 \\ -0.1689 & -0.4389 \end{pmatrix}$
$(\mathbf{B}+\mathbf{VD})n_0$ (mm)	$\begin{pmatrix} 14.5787 & -0.2592 \\ -0.2833 & 16.1480 \end{pmatrix}$	$\begin{pmatrix} 14.2923 & -0.2545 \\ -0.2785 & 15.8283 \end{pmatrix}$
$\mathbf{e}+\mathbf{V}\boldsymbol{\pi}$ (mm)	$\begin{pmatrix} -0.2762 \\ 0.1950 \end{pmatrix}$	$\begin{pmatrix} -0.2767 \\ 0.1950 \end{pmatrix}$
	$\Delta((\mathbf{B}+\mathbf{VD})n_0) = \begin{pmatrix} -0.2864 & 0.0047 \\ 0.0048 & -0.3197 \end{pmatrix}$ mm $\Delta(\mathbf{e}+\mathbf{V}\boldsymbol{\pi}) = \begin{pmatrix} -0.0006 \\ 0.0000 \end{pmatrix}$ mm	
	$\Delta\mathbf{Z} = \begin{pmatrix} -0.2889 & 0.0042 \\ 0.0042 & -0.3230 \end{pmatrix}$ mm (Eq. 5) $\Delta\mathbf{y} = \begin{pmatrix} -0.0008 \\ 0.0160 \end{pmatrix}$ mm (Eq. 29)	

$\Delta\mathbf{F}_0 = \mathbf{F}_{0b} - \mathbf{F}_{0r} = \begin{pmatrix} -1.1136 & -0.0298 \\ -0.0298 & -0.9565 \end{pmatrix}$ D which is $-0.9510 \quad -0.1680 \times 100.37$ as a spherocylindrical power. It follows from the definitions that there is no simple relationship between chromatic difference of refractive compensation and longitudinal chromatic aberration for a distant object point.

Near Object Point

Table A3 lists the details for the model eye and object point O 400 mm in front of the eye and with transverse position $\mathbf{y}_{\mathrm{O}} = \begin{pmatrix} -30 \\ 30 \end{pmatrix}$ mm relative to longitudinal axis Z. For an observer looking at the eye along Z, with O between the observer and the eye, O is up and to the left. The principal structure of the longitudinal chromatic aberration is -0.3555 mm along $97.25°$ and -0.3139 mm. The principal longitudinal chromatic aberrations are slightly larger in magnitude compared with those for the distant object and the principal meridians have undergone a small anticlockwise rotation. The horizontal and vertical components of the transverse chromatic aberration are -0.0238 mm and 0.0262 mm.

REFERENCES

1. Villegas ER, Carretero L, Fimia A. Le Grand eye for the study of ocular chromatic aberration. Ophthal Physiol Opt 1996; 16: 528-31.
2. Powell I. Lenses for correcting chromatic aberration of the eye. Appl Opt 1981; 20: 4152-5.
3. Harris WF. Transferences of heterocentric astigmatic catadioptric systems including Purkinje systems. Optom Vis Sci 2010; 87: 778-86.
4. Harris WF. A unified paraxial approach to astigmatic optics. Optom Vis Sci 1999; 76: 480-99.

TABLE A3.

Longitudinal $\Delta\mathbf{Z}$ and transverse $\Delta\mathbf{y}$ chromatic aberration of a model heterocentric astigmatic eye and object point with $z_O = -400$ mm and $\mathbf{y}_O = \begin{pmatrix} -30 \\ 30 \end{pmatrix}$ mm.

	red	blue
$\mathbf{L}$ (kD) (Eq. 10)	$\begin{pmatrix} -0.5773 & -0.2359 \\ -0.2359 & 1.9984 \end{pmatrix}$	$\begin{pmatrix} -0.4875 & -0.4410 \\ -0.4410 & 4.3566 \end{pmatrix}$
$\mathbf{Z}$ (mm) (Eq. 4)	$\begin{pmatrix} -2.2038 & -0.2601 \\ -0.2601 & 0.6367 \end{pmatrix}$	$\begin{pmatrix} -2.5184 & -0.2549 \\ -0.2549 & 0.2818 \end{pmatrix}$
z_- (mm)	-2.2275	-2.5414
z_+ (mm)	0.6603	0.3048
$\mathbf{v}_+$	$\begin{pmatrix} 0.0904 \\ -0.9959 \end{pmatrix}$	$\begin{pmatrix} 0.0899 \\ -0.9959 \end{pmatrix}$
$\mathbf{V}$ (mm) (Eq. 26)	$\begin{pmatrix} -1.6524 & -0.1950 \\ -0.1950 & 0.4774 \end{pmatrix}$	$\begin{pmatrix} -1.8792 & -0.1902 \\ -0.1902 & 0.2103 \end{pmatrix}$
$\mathbf{A}+\mathbf{VC}$	$\begin{pmatrix} -0.0377 & 0.0007 \\ 0.0008 & -0.0419 \end{pmatrix}$	$\begin{pmatrix} -0.0369 & 0.0007 \\ 0.0008 & -0.0410 \end{pmatrix}$
$\mathbf{e}+\mathbf{V}\boldsymbol{\pi}$ (mm)	$\begin{pmatrix} -0.2860 \\ 0.2011 \end{pmatrix}$	$\begin{pmatrix} -0.2864 \\ 0.2009 \end{pmatrix}$
	$\Delta(\mathbf{A}+\mathbf{VC}) = \begin{pmatrix} 0.0008 & -0.0000 \\ -0.0000 & 0.0009 \end{pmatrix}$ $\Delta(\mathbf{e}+\mathbf{V}\boldsymbol{\pi}) = \begin{pmatrix} -0.0004 \\ -0.0001 \end{pmatrix}$ mm	
	$\Delta\mathbf{Z} = \begin{pmatrix} -0.3145 & 0.0052 \\ 0.0052 & -0.3548 \end{pmatrix}$ mm (Eq. 5) $\Delta\mathbf{y} = \begin{pmatrix} -0.0238 \\ 0.0262 \end{pmatrix}$ mm (Eq. 28)	