Appendix.

An alternative method of deriving the paraxial expression is here.

For a thin prism in air with apex angle α, if light enters the prism at an angle of incidence approaching zero then the refraction can be considered to occur at the rear surface of the prism alone.

Snell's Law yields the equation (1)

i.e. $n_2 \sin\alpha = \sin(\delta_a + \alpha)$

where δ_a represents the deviation of the prism when in air.

If the paraxial approximation of $\sin\alpha = \alpha$ is applied where α is in radians, then equation(1) simplifies to

$$n_2\alpha = \delta_a + \alpha.$$

This can be re-arranged to give

$$\delta_a = \alpha(n_2 - 1)$$

Let the thin lens now be immersed in solution(n_1).

As was the case previously, the only significant refraction will occur at the back surface of the prism.

Applying Snell's Law at the back surface of the prism;

$n_2 \sin\alpha = n_1(\sin\delta_w + \alpha)$ where δ_w is the deviation of the light beam in solution.

Using paraxial approximations, the equation simplifies to

$$n_2\alpha = n_1(\delta_w + \alpha) \qquad (11)$$

Re-arranging, this becomes

$$\delta_w = \alpha(n_2 - n_1)/n_1 \qquad (12)$$

When the light exits from the solution and is refracted at the boundary with air (the glass interface being ignored), refraction will again occur.

Snell's Law yields

$$n_3 \sin\delta_{ia} = n_1 \sin\delta_w \qquad (13)$$

where δ_{ia} is the deviation as measured in air that the immersed prism has caused and $n_3 = 1$.

This can be simplified to $\delta_{ia} = n_1\delta_w$.

Hence the deviation(δ_{ia}) when a prism is immersed in a solution(n_1) is

$$\delta_{ia} = n_1\alpha(n_2 - n_1)/n_1$$

$$= \alpha(n_2 - n_1) \qquad (14)$$

So the ratio of (deviation when prism is immersed(δ_{ia}))/(deviation in air(δ_a)) is

$$(n_2 - n_1)/(n_2 - 1) \qquad (15)$$

This is identical to the equation found previously[1] for lens power immersed in solution and in air.

Equation(15) can be re-arranged to provide an expression for n_2.

$$n_2 = (n_1\delta_a - \delta_{ia})/(\delta_a - \delta_{ia}). \qquad (16)$$

In the realm of paraxial optics where angles are small, the paraxial approximation $\sin\delta = \delta$ can be applied if δ is in radians. If $\sin\delta = \delta$ is valid, then a similar approximation $\tan\delta = \delta$ can be applied .

Hence, $\Delta/100 = \delta$.

So the formula $n_2 = (n_1\delta_a - \delta_{ia})/(\delta_a - \delta_{ia})$ can be re-written in terms of prismatic power measured with a lensmeter. Say the prismatic power measured in air is Δ_1 and the prismatic power measured by the lensmeter when the prism(or lens) is immersed in water is Δ_2.

Then since $\delta_a = \Delta_1/100$ and $\delta_{ia} = \Delta_2/100$,

$$n_2 = (n_1\Delta_1 - \Delta_2)/(\Delta_1 - \Delta_2) \qquad (17)$$

So the value of refractive index can be calculated using this equation by simply measuring the prismatic power of the lens in air and then in a second medium such as water.