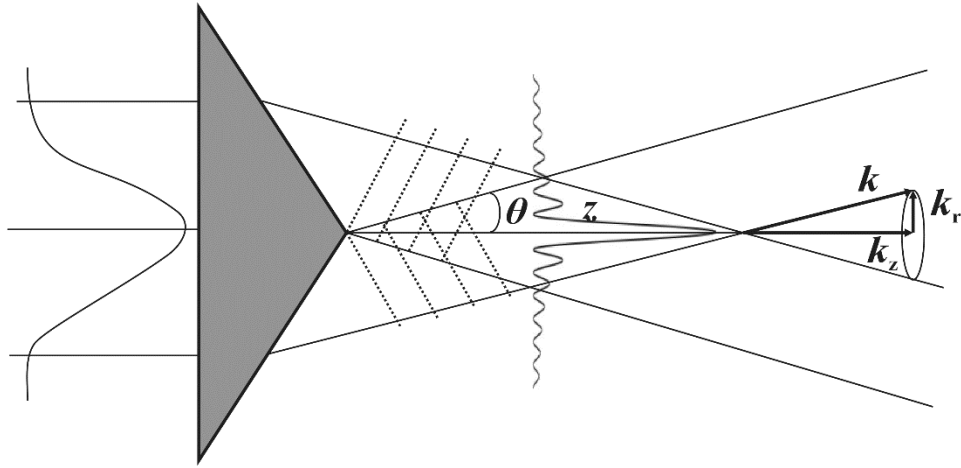


## APPENDIX

Bessel beams, formed by superposition of a set of plane waves that undergo the same phase shift over a spatial propagation, with the wave vectors propagating on a cone (Fig. A1), are localized beams with transverse patterns that remain stationary along the propagation distance. The electric field component of a Bessel beam can be represented as<sup>7</sup>

$$E(r, \phi, z) = A_0 \exp(ik_z z) J_n(k_r r) \exp(\pm im\phi)$$

where  $J_n$  is an  $n$ th-order Bessel function of the first kind,  $A_0$  is the amplitude of the electric component of the electromagnetic wave,  $k_z$  is the longitudinal wave vector,  $k_r$  is the radial wave vector, and  $r$ ,  $\phi$  and  $z$  are the radial, azimuthal and longitudinal components, respectively. This equation may be reduced by  $|J_n|$  as when generated by an axicon or similar. The phase carries the important properties, from which the spiral vortex set by order  $m$  may also be omitted.



**Figure A1.** Spatial spectrum of a Bessel beam generated using an axicon with wave vectors of plane waves on the surface of a cone.  $\theta$ , Opening angle of cone;  $k$ , wave vector;  $k_r$ , radial component of  $k$ ;  $k_z$ , longitudinal component of  $k$ . (Modified from reference<sup>6</sup>)

The transportation of energy from rings of the Bessel beam prevents the central lobe from spreading as well as contributing to the beam reconstruction beyond an obstruction,<sup>7, 11, 24</sup> i.e., if an obstructing object is placed in the center of the beam, the energy from the rings is transported to the center of the beam and the initial profile is reformed after a further distance  $z_{\min}$  given by<sup>7</sup>

$$z_{\min} \approx \frac{ak}{2k_z}$$

where  $a$  is the width of the obstruction measured from the beam center and  $k$  is the wave number whose magnitude is given by

$$k = \sqrt{k_z^2 + k_r^2}$$