APPENDIX

Derivation of Equation (1)

The derivation of eq. (1) follows closely the derivation given by Bennett (ref. 2, "Emsley and Swaine's Ophthalmic Lenses-Vol. 1", pages 205-206, 208-209).

In Fig. A1, *O* is a point on the optical axis and a distance *l* from a thin lens of power *F*. *Q* is an object point at a height *h* from *O*. Lens optical center *A* is along the line of sight when the eye is looking straight ahead. The image point *Q'* in the lens becomes the object for the eye; it is a height *h'* from the point *O'* on the optical axis. To look at *Q'*, the eye must turn about its center-of-rotation at *C*, a distance *s* from the lens, so that the line of sight is now along the line *Q'C* and intercepts the lens at point *V*, which is at a height *v* from the object axis.

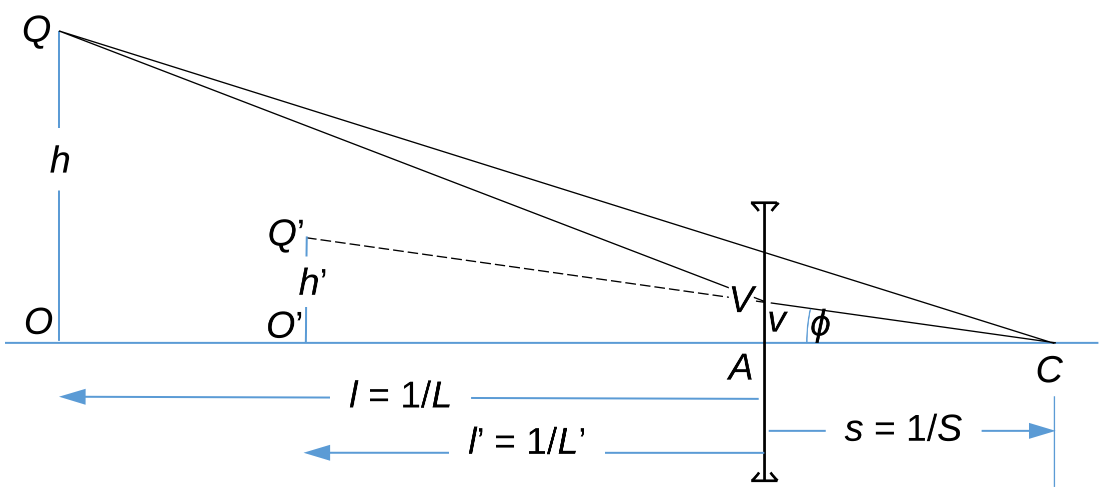

Figure A1. Derivation of paraxial equations for ocular rotation through a lens.

From similar triangles in Fig. A1

$$AV/AC = O'Q'/OC = O'Q'/(O'A + AC)$$

or

$$v/s = h'/(-l' + s)$$

from which

$$v = sh'/(-l' + s)$$

Replacing distances by the corresponding dioptral terms, i.e. l by the object vergence L, l' by the image vergence L', and s by S, gives

$$v = -h'L'/(S - L')$$

Since $h'L' = hL$ and $L' = L + F$, this becomes

$$v = -hL/(S - L - F)$$

The angle ϕ through which the eye turns from the straight ahead direction to look at Q' can be given in prism dioptres by $100v/s$. Accordingly,

$$\phi \text{ (in prism diopters)} = 100vS = -100hLS/(S - L - F) \qquad \text{(A1)}$$

This is similar to Bennett's eq. (13.2).

A prism by itself would form an image of Q' in line with Q and O at Q', with the original height altered by q (Fig. A2). Its effect on ocular rotation u (also effective prism power) is, similar to eq. (A1),

$$u' = -100qLS/(S - L - F) \qquad \text{(A2)}$$

As the angular deviation produced by the prism is $-q/l$ radians, its prismatic power in prism diopters is

$$u' \text{ (in prism diopters)} = -100qL$$

Substituting P for $-100qL$ in eq. (A2) gives

$$u' \text{ (in prism diopters)} \approx PS/(S - L - F) \qquad \text{(A3)}$$

which is Bennett's eq. (13.6) except for the use of u' rather than $\Delta\theta$.

To be consistent with the sign convention in this paper, a change of sign is required to give

$$u' \text{ (in prism diopters)} \approx -PS/(S - L - F) \qquad (1)$$

For a plano prism, $F = 0$, and the equation would be

$$u' \text{ (in prism diopters)} \approx -PS/(S - L) \qquad \text{(A4)}$$

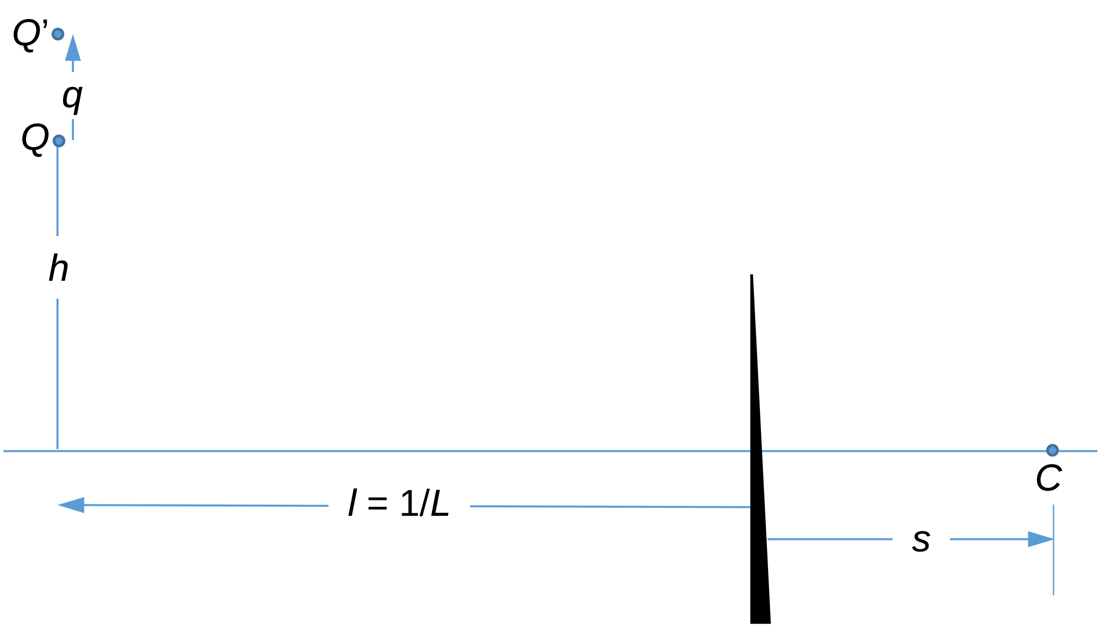

Figure A2. Relating to the determination of the component of eye rotation due to a plano prism.

Bennett gave an alternate derivation of eq. (A4). In Fig. A3, C is again the center of rotation of the eye, the axial object point is at O, d is the angle of deviation at the prism, the raypath intercept of the prism is at V, and the angle of eye rotation is u'. l and s are the distances from the prism to the object and to the center of rotation of the eye, respectively. The image point *O'* formed by the prism is displaced a distance *q* from *O*. In radians,

$$d = -q/l$$

and hence

$$q = -dl$$

and

$$u' = q/(-l + s) = -dl/(-l + s) = dS/(S - L)$$

In prism diopters, *d* becomes the prism power *P* to give

$$u' \text{ (in prism diopters)} = PS/(S - L)$$

To be consistent with the sign convention in this paper,

$$u' \text{ (in prism diopters)} = -PS/(S - L) \qquad \text{(A4)}$$

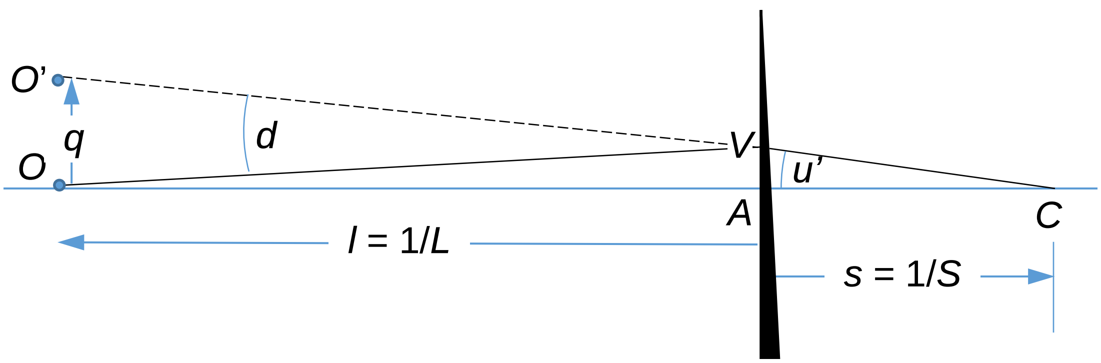

Figure A3. Relationship between ocular rotation and deviation of a plano prism in near vision.