Improved Liver Allocation with Optimized Neighborhoods
Technical Appendix

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The neighborhood optimization model forms neighborhoods around each DSA with certain desired properties. The neighborhood of a DSA identified from the optimization model forms this DSA's region in a local-regional-national policy. For the results presented in the article, this is done so that the combined DSA neighborhood organ need (demand) and organ availability (supply) ratio is as close as possible to the national organ need and availability ratio, while other structural requirements (described below) are ensured. We emphasize that the output from the model is a neighborhood structure to be used in the local-regional-national framework, i.e., *neither the optimization model nor its output specify the number of organs an OPO could be sending to its region.*

We now provide the mathematical details of the neighborhood approach to achieve greater equity and efficiency in the liver allocation system.

1. Mathematical Formulation of the Neighborhood Model

Let $I = \{1, 2, ...n\}$ be the set of DSAs and $\{1, 2, ...T\}$ be the years in the study period. We formulate a stochastic program that minimizes the expected disparity in organ supply and demand over T years. Moreover, we assume that I carries with it an adjacency matrix that identifies whether $i \in I$ and $j \in I$ share physical boundaries.

1.1. Decision Variables. The binary decision variables are x_{ij}, which take the value 1 if DSA $j \in I$ is linked to DSA $i \in I$.

1.2. Parameters. Let $s_i(t)$ be the number of livers recovered in DSA i in year t; Let $d_i(t)$ be the number of organs demanded in DSA i in year t. We measure $d_i(t)$ as the number of waitlist additions in DSA i in year t; Let c_i and p_i be the number of active transplant centers and population respectively in DSA i; $\underline{C}$ is the minimum allowable number of transplant centers that can be in a neighborhood; $\underline{P}$ is the minimum allowable population in a neighborhood; $\underline{M}$ and $\bar{M}$ are lower and upper bounds respectively on the number of DSAs that can be in a neighborhood. Let θ be the target for the ratios of neighborhood supplies and demands to be achieved. Specifically, we take $\theta = \frac{1}{T}\sum_{t=1}^{T} \frac{\sum_{j=1}^{n} d_j(t)}{\sum_{j=1}^{n} s_j(t)}$, that is, θ is the expected value of the ratio of nationally aggregated organ supply and demand, where the expectation is taken over T years.

Define v_{ij} to be the historical volumes of organs procured in DSA i and transplanted in DSA j and similarly define τ_{ij} to be the historical average transport distances (or times) of organs procured in DSA i and transplanted in DSA j. Let $\bar{\tau}$ be the desired bound for average organ-transport distance or time.

1.3. Neighborhood Structure. In order to impose a specific type of contiguity and geographic immediacy requirements, for each DSA i, we define a relation such that for each $j, k \in I$, $j \prec_i k$ if and only if the minimum number of adjacent DSAs required to traverse from DSA i to DSA j is less than the minimum number of adjacent DSAs required to traverse from DSA i to DSA k.

1.4. **Objective.** The objective is to minimize the expected value of the maximum absolute deviation of the ratios of the number of organs available to a DSA's neighborhood (not a specific DSA) and the number of organs needed in a DSA's neighborhood from the target value, the national ratio θ. In the specific output used for the results presented in the article, the number of waitlist additions each year is used to estimate organ need.

1.5. **Description of Constraints.** The constraints enforce that a DSA has a minimum and a maximum number of neighbors that it can be linked to (Density); the average volume-weighted transport distance or time in a neighborhood is bounded (Compactness); neighborhoods are contiguous and DSAs that are geographically immediate to a given DSA are in its neighborhood (Contiguity); neighborhoods have a minimum number of transplant centers and population (Centers; Population respectively); each DSA is in its own neighborhood (Reflexivity); and DSA i is in DSA j's neighborhood if and only if DSA j is in i's neighborhood (Symmetry).

(Neighborhoods)

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\min\limits_{x_{ij}} & \dfrac{1}{T}\displaystyle\sum_{t=1}^{T}\max_{i\in I}\left|\frac{\sum_{j=1}^{n} d_j(t)x_{ij}}{\sum_{j=1}^{n} s_j(t)x_{ij}}-\theta\right| & \\
\text{s.t.} & \underline{M}\leq\displaystyle\sum_{j=1}^{n} x_{ij}\leq\bar{M} & \forall i\in I \ \textit{(Density)} \\
 & \displaystyle\sum_{j=1}^{n}(\tau_{ij}-\bar{\tau})v_{ij}x_{ij}\leq 0 & \forall i\in I \ \textit{(Compactness)} \\
 & x_{ij}\geq x_{ik} & \forall i,j,k\in I \text{ with } j\prec_i k \ \textit{(Contiguity)} \\
 & \displaystyle\sum_{j=1}^{n} c_j x_{ij}\geq\underline{C} & \forall i\in I \ \textit{(Centers)} \\
 & \displaystyle\sum_{j=1}^{n} p_j x_{ij}\geq\underline{P} & \forall i\in I \ \textit{(Population)} \\
 & x_{ii}=1 & \forall i\in I \ \textit{(Reflexivity)} \\
 & x_{ij}=x_{ji} & \forall i,j\in I \ \textit{(Symmetry)} \\
 & x_{ij}\in\{0,1\} & \forall i,j\in I \ \textit{(Integer)}
\end{array}
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