Mathematical steps to calculate ECBF, knowing Hb, SaO_2, CO and VO_2

We start from the two following equations describing the steady-state:

$$\begin{cases} S_vO_2 = S_aO_2 * 1 - \dfrac{VO_2}{DO_2} \\ S_aO_2 * Q = S_{ECMO}O_2 * ECBF + S_vO_2 * (Q - ECBF) \end{cases}$$

$$\begin{cases} S_vO_2 = S_aO_2 - \dfrac{VO_2}{k * Hb * 10 * Q} \\ S_vO_2 * (Q - ECBF) + S_{ECMO}O_2 * ECBF = S_aO_2 * CO \end{cases}$$

By substituting the first equation into the second the following relationship is obtained:

$$\left(S_aO_2 - \frac{VO_2}{k * Hb * 10 * Q}\right) * (Q - ECBF) + S_{ECMO}O_2 * ECBF = S_aO_2 * Q$$

The following mathematical steps are required to solve this equation for S_aO_2:

$$(S_aO_2 * k * Hb * 10 * Q - VO_2) * (Q - ECBF) + S_{ECMO}O_2 * k * Hb * 10 * Q * ECBF = S_aO_2 * k * Hb * 10 * Q * Q$$

Dividing each term by Q:

$$\left(S_aO_2 * k * Hb * 10 - \frac{VO_2}{Q}\right) * (Q - ECBF) + S_{ECMO}O_2 * k * Hb * 10 * ECBF = S_aO_2 * k * Hb * 10 * Q$$

$$S_aO_2 * k * Hb * 10 * Q - S_aO_2 * k * Hb * 10 * ECBF - VO_2 + \frac{VO_2 * ECBF}{Q} + S_{ECMO}O_2 * k * Hb * 10 * ECBF = S_aO_2 * k * Hb * 10 * Q$$

Removing the identical terms in the two sides of the equation:

$$S_aO_2 * k * Hb * 10 * ECBF = S_{ECMO}O_2 * k * Hb * 10 * ECBF + \frac{VO_2 * ECBF}{Q} - VO_2$$

Rearranging the equation:

$$ECBF * \left[(k * Hb * 10) * (S_aO_2 - S_{ECMO}O_2) - \frac{VO_2}{Q}\right] = -VO_2$$

And finally solving for ECBF and assuming $S_{ECMO}O_2 = 1$ we get to equation 5

$$ECBF = \frac{VO_2}{\frac{VO_2}{Q} + (k * Hb * 10) * (1 - S_aO_2)} \quad (5)$$

Mathematical steps to calculate ECBF, knowing Hb, CO, S_vO_2 and VO_2

We start from the two following equations describing the steady-state:

$$\begin{cases} S_vO_2 = S_aO_2 * 1 - \dfrac{VO_2}{DO_2} \\ S_aO_2 * Q = S_{ECMO}O_2 * ECBF + S_vO_2 * (Q - ECBF) \end{cases}$$

$$\begin{cases} S_vO_2 = S_aO_2 - \dfrac{VO_2}{k * Hb * 10 * Q} \\ S_aO_2 * Q = S_{ECMO}O_2 * ECBF + S_vO_2 * (Q - ECBF) \end{cases}$$

$$\begin{cases} S_aO_2 = S_vO_2 + \dfrac{VO_2}{k * Hb * 10 * Q} \\ S_aO_2 * Q = S_{ECMO}O_2 * ECBF + S_vO_2 * (Q - ECBF) \end{cases}$$

By substituting the first equation into the second the following relationship is obtained:

$$\left(S_vO_2 + \frac{VO_2}{k * Hb * 10 * Q}\right) * CO = S_{ECMO}O_2 * ECBF + S_vO_2 * Q - S_vO_2 * ECBF$$

The following mathematical steps are required to solve this equation for ECBF:

$$S_vO_2 * Q + \frac{VO_2}{k * Hb * 10} = S_{ECMO}O_2 * ECBF + S_vO_2 * Q - S_vO_2 * ECBF$$

Removing the identical terms in the two sides of the equation:

$$\frac{VO_2}{k * Hb * 10} = S_{ECMO}O_2 * ECBF - S_vO_2 * ECBF$$

And assuming $S_{ECMO}O_2 = 1$

$$\frac{VO_2}{k * Hb * 10} = ECBF - S_vO_2 * ECBF$$

And finally solving for ECBF, we get to equation 6:

$$ECBF = \frac{VO_2}{k * Hb * 10 * (1 - S_vO_2)} \quad (6)$$