## Supplementary Online Material.

## eAppendix A: Applications using the generic graph in Figure 6.

In eFigure 1 and 2 we "zoom in" on the relevant area on Figure 6 for the particular application plotting all countries from the health economic example in the main text,<sup>3</sup> and an additional epidemiological application from the well-cited Mackenbach et al.<sup>1</sup> That is, we plot the absolute inequality against mean health (i.e. survival and mortality rates), measured as shortfalls (e.g. death rates) from right to left and the matching attainments (e.g. survival rates) from left to right. The upward sloping red lines represent populations with a particular level of attainment-relative inequality but varying mean. Analogously, the downwards sloping blue lines represent populations with a particular level of shortfall-relative inequality but varying mean. In the general graph, Figure 6, these are represented as lines starting at the attainment origin and shortfall origin, respectively (proportionally reducing the level of attainment/shortfall to zero gradually reduces both the absolute inequality and the average level of attainment/shortfall to zero). Any point above a line represents a higher level of inequality, and any point below a line represents a lower level of inequality. Thus, the steeper the ray, the higher the level of inequality. As mentioned in the main text, the location on the general graph affects the slope of the lines representing the shortfall-relative and attainmentrelative inequality. For all three examples, the relevant area is at different locations in the bottom right corner which is why the attainment-relative lines are not as steep as the shortfallrelative ones in all three cases.

Although we do not draw the lines representing the inequality levels in any particular country, we may come to similar conclusions as for Figure 4 and Figure 5 using these generic lines as guides. For example, in eFigure 1 we can see by the help of the generic lines that while absolute and attainment-relative inequality is higher in Luxemburg than in Denmark (the red line below the point representing Luxemburg is above the point representing Denmark), shortfall-relative inequality is higher in Denmark (i.e., the blue line is below the point representing Denmark but above the point representing Luxemburg). Even without lines crossing between the countries making the comparison obvious, we can draw similar conclusions only using the slope of generic lines as a guide: looking specifically at Austria and Belgium we can see that while absolute and attainment-relative inequality is higher in Austria than in Belgium, shortfall-relative inequality is higher in Belgium.

The graphs also directly show the relationship between the value judgments and the mean health, illustrating how the three measures provide an ethical benchmarks against which to judge the difference in inequality between populations. Even without lines representing the level of inequality in the particular countries, the graph provides an intuitive view on how, for example, the outliers Portugal (eFigure 1) and Lithuania (eFigure 2) would have to distribute improvements in attainment in order to reach similar levels of absolute inequality as the other countries. Thus, for a larger set of countries with an interest broader than one specific country a graph using only generic lines still provides information about the nature of the inequality.

To consider the results for the 16 European countries from Mackenbach et al.,<sup>1</sup> we have to "zoom in" on the *very* bottom right of Figure 6, yielding close to horizontal attainmentrelative lines. The flat slope of the red lines in eFigure 2 illustrates why the attainmentrelative inequality rankings in this case are going to be very close to the absolute inequality rankings. The graph also shows why using a shortfall-relative measure is going to change the inequality story a lot for some countries (e.g. France and Estonia) in this application. Again, note that for applications using variables and populations with different means these slopes will be different.



eFigure 1: Socioeconomic Inequality in Self-Assessed Health in Europe

Note eFigure 1 plots the absolute inequality as measured by the generalized concentration index against the mean attainment/shortfall of self-assessed health. Along the blue lines the level of shortfall-inequality is the same and along the red levels of shortfall-relative inequality is the same. Any point above a line represents populations with a higher level of inequality than a population on or below the same line. To obtain the attainment-relative lines plot the generalized concentration index as a function of the mean using the level of (attainment/shortfall-relative) constant: generalized concentration index.



eFigure 2: Socioeconomic Inequality in Survival/Mortality in Europe

Note eFigure 2 plots the absolute inequality as measured by the SII against the mean survival/mortality rate per 10000 for the 16 countries, or 19 populations, presented in Mackenbach et al.<sup>1</sup> Observe that for Spain the level of inequality is reported for three populations from different regions, Barcelona (Spain B), Madrid (Spain M), and the Basque Country (Spain BC). Along the blue lines the level of shortfall-inequality is the same and along the red levels of shortfall-relative inequality is the same. Any point above a line represents populations with a higher level of inequality than a population on or below the same line.

## eAppendix B: Many more possible health inequality measures?

The attainment-relative, absolute, or shortfall-relative measures only represent three different perspectives on what change would constitute an inequality preserving one but we could also consider many more. Some people may believe that an inequality preserving change might coincide with a change induced by a mixture of the red and the blue injections: that is, all individuals in the population take a fraction of the red injection yielding a fractional uniform increase and a fraction of the blue injection yielding a fractional proportional increase in attainments. In the income inequality literature, this mixture is referred to as an intermediate, or a compromise between the absolute and the (attainment-)relative measure.<sup>38</sup> Positions outside of these polar cases may be considered to be incompatible with a plausible view. For example, those inequality equivalence criteria where the level of inequality will increase for a uniform improvement even though absolute differences (in attainment) stay constant and relative differences decrease (such inequality equivalence criteria are referred to as extreme leftist or inverse-relative).<sup>17,40</sup>

For a bounded health variable, we can think of a similar concept for shortfalls of health; some people may believe that an inequality preserving change might coincide with a change induced by a mixture of the yellow and green injections: that is, a compromise between the absolute and the shortfall-relative. In terms of attainment, however, all inequality equivalence criteria represented by combinations of the yellow and the green (except for the green itself) lies outside the positions that would be judged as reasonable if applying the income inequality concept: The change induced by the green injection decreases both relative and absolute differences in attainment (but relative differences in shortfalls are kept constant). As these inequality equivalence criteria appears ok from the perspective of shortfall, it highlights the question if the concepts from the income inequality literature are necessarily relevant in a context of bounded health variables. In support of such an argument, Allanson and Petrie<sup>13,14</sup> and Kjellsson and Gerdham<sup>34</sup> stress that the range of inequality equivalence criteria that are ethically defensible may be different for bounded and unbounded variables. Possibly, the inequality equivalence criteria that correspond to the proportional changes induced by the blue and green injections are the relevant polar cases of the range of sensible inequality equivalence criteria for a bounded variable. In other words, such a compromise says that the proportional increase of life expectancy induced by the blue injection does not decrease inequality and that the proportional decrease of shortfalls of life expectancy induced by the green injection does not increase inequality (compare Figure 3).

The range of indices that corresponds to a mixture of the blue and the green injections, that is a compromise between an attainment-relative and shortfall-relative, includes several known indices where noteworthy the absolute index is not the only yielding consistent rankings. Among these index are the adjustment of the concentration index suggested by Guido Erreygers<sup>15</sup> and Adam Wagstaff<sup>31</sup> which been intensely debated in the literature <sup>16–22</sup>: they both yield consistent rankings for attainments and shortfalls, but there are differences in the underlying value judgments. While the Erreygers index is an absolute measure, the inequality equivalence criteria imbedded in the Wagstaff index is more complex and depends on the mean. The latter index is included in a wider family of indices that combines relative inequality in shortfalls and attainments into one single measure.<sup>34,39</sup> All these measures could also be plotted as curves that represent different levels of inequality on a graph such as Figure 4, 5 and 6. These curves will be a combination of the shortfall-relative and the attainment-relative lines.

The current empirical research of individuals' perception of inequality preserving changes does not support the strict preference of any particular measure over another. The additional complexity that these compromise measures introduce may therefore currently outweigh the additional benefits of presenting these in addition to the less complicated and easier to describe measures, especially as they all fall between the shortfall-relative and the attainment-relative measure.