

Figure S1: Seasonal Variation Introduces Bias. We plot the estimated values of $\hat{\theta}$ for a range of values of θ , under various seasonal conditions. Seasonal disease of interest but not negative control is plotted in pink. Inverse seasonality of the two events is plotted in blue, with the larger seasonal effect shown using a dotted line. The orange line shows a case when the two seasonal effects have different periods. To implement seasonality, the base hazard was multiplied by a seasonal effect function $f_P(t)$ or $f_N(t)$. Inverse seasonality (shown in pink) represented by $f_P(t) = \sin(2\pi t)$ and $f_N(t) = \cos(2\pi t)$. Larger effects (dotted line) were investigated using $f_P(t) = 3\sin(2\pi t)$ and $f_N(t) = 3\cos(2\pi t)$. We implemented seasonality with different periods (shown in oranges) with $f_P(t) = \sin(\pi t)$ and $f_N(t) = \cos(2\pi t)$. We also investigated only one event having seasonal effects (shown in pink) with $f_P(t) = \sin(2\pi t)$ and $f_N(t) = 1$.

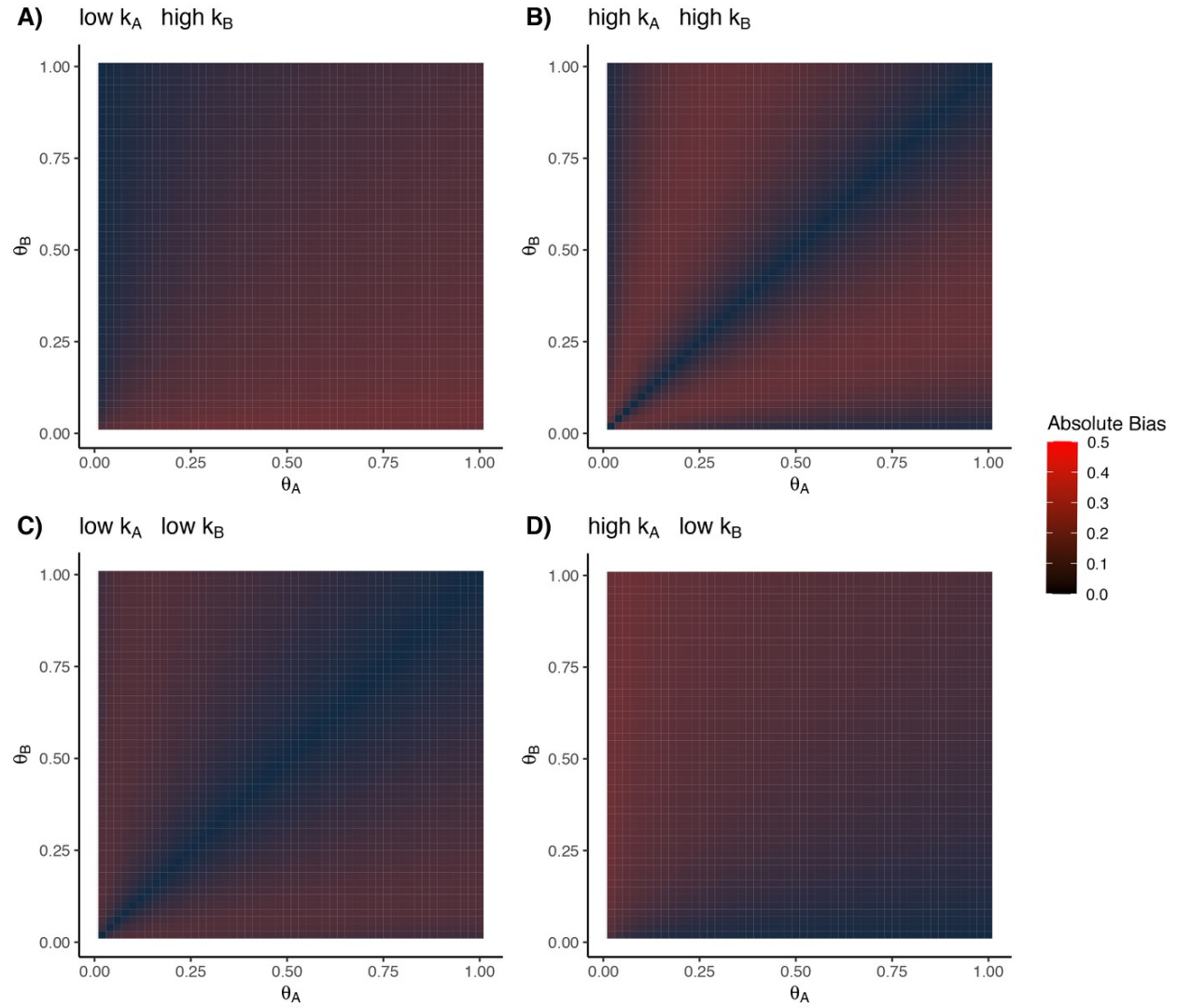
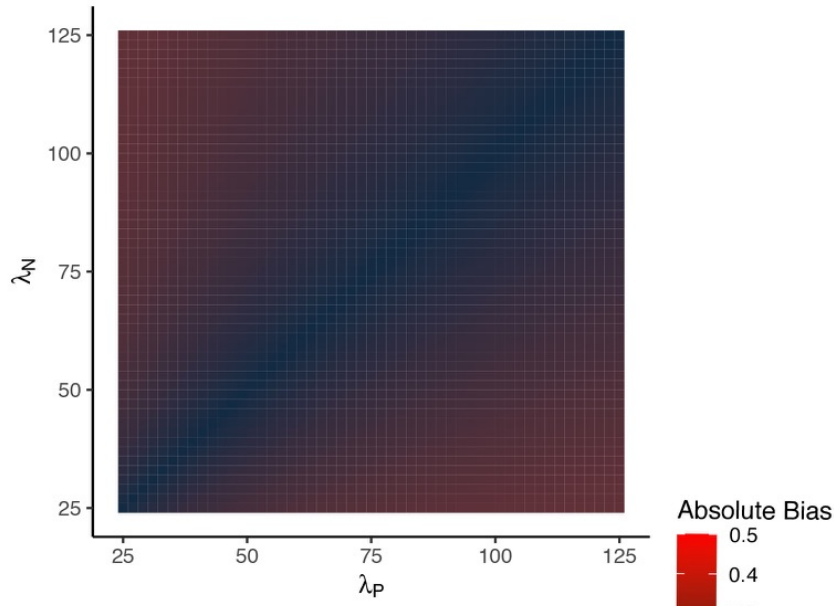


Figure S2: Bias of Gamma Distributed Event Time Assumption. Here we plot the absolute difference between $P(A_i < B_i)$ for the exponential event times and gamma event times. Larger discrepancies are shown as more red, and greater concordance as more black. We show four pairs of shape where k_A and k_B are either low (equal to 0.5) or high (equal to 3) in the four panels. Each cell is filled according to the absolute value of the difference between $\frac{k_A \theta_A}{k_A \theta_A + k_B \theta_B}$ and the regularized incomplete beta function

$$IB\left(\frac{\theta_A}{\theta_A + \theta_B}, k_B, k_A\right).$$

A) $k = 0.5$



B) $k = 2$

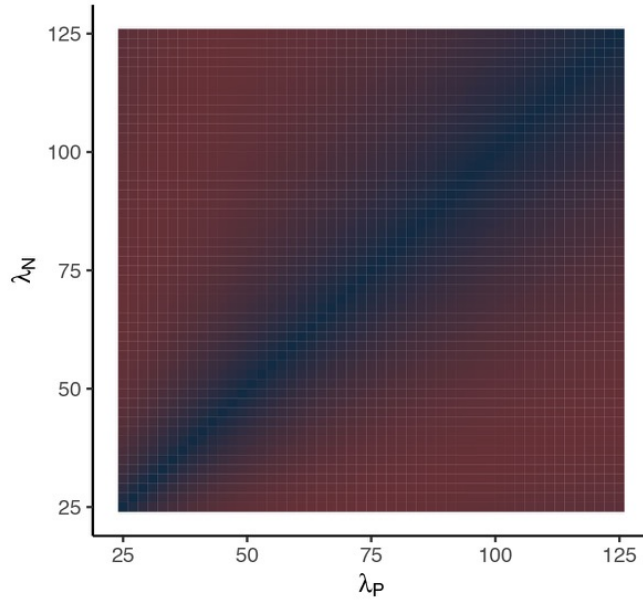


Figure S3: Bias of Weibull Distributed Event Time Assumption. Here we plot the absolute difference between $P(A_i < B_i)$ for the exponential event times and Weibull event times. Larger discrepancies are shown as more red, and greater concordance as more black. In A) the shape of the Weibull distributions $k = 0.5$, and in B) $k = 2$. Cells are filled according to the absolute difference between $\frac{\lambda_P}{\lambda_P + \lambda_N}$ and $1/1 + (\frac{\lambda_N}{\lambda_P})^k$

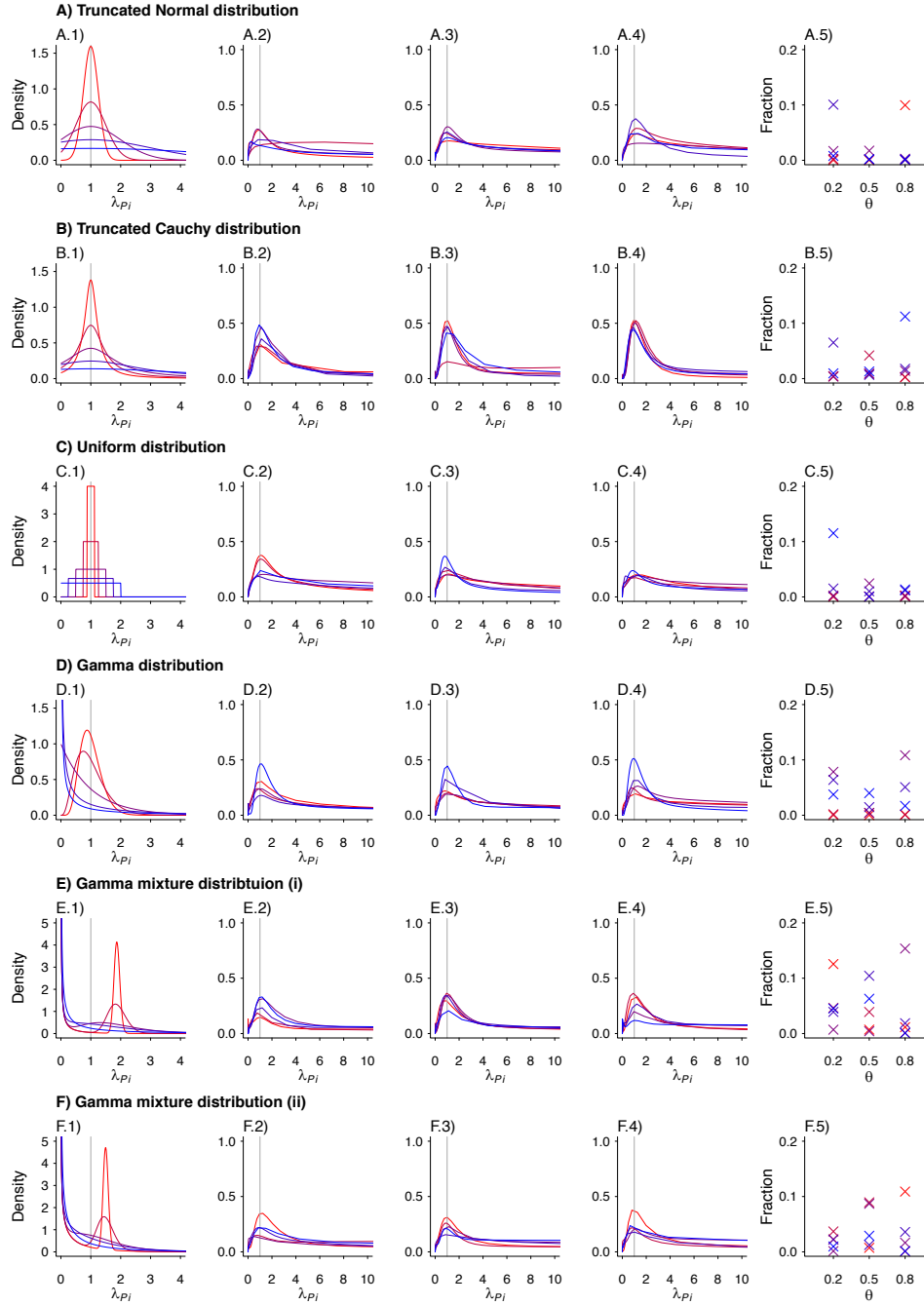


FIGURE S4: Estimated Density Kernels For Fixed Effects Model. We plot the estimated density kernels for the individual effects estimated by the fixed effects model for each individual in a simulated study. In each row, column 1 is a reproduction of the densities used to produce the individual λ_{pi} . Columns 2-4 are the estimated density kernels for $\theta = 0.2$, $\theta = 0.5$, $\theta = 0.8$, respectively. Column 5 shows the proportion of estimates which were over 1000 for each set of parameters. Values are plotted on a red-to-blue color ramp corresponding to the parameterizations I-V, respectively, in order of least (I; red) to greatest (V; blue) variance as detailed in Table 4. **A)** Truncated Normal distribution; **B)** Truncated Cauchy distribution; **C)** Uniform distribution; **D)** Gamma distribution; **E)** Mixture of Gamma distributions (i) with means at 0.125 and 1.875; and **F)** Mixture of Gamma distributions (ii) with means at 0.5 and 1.5.

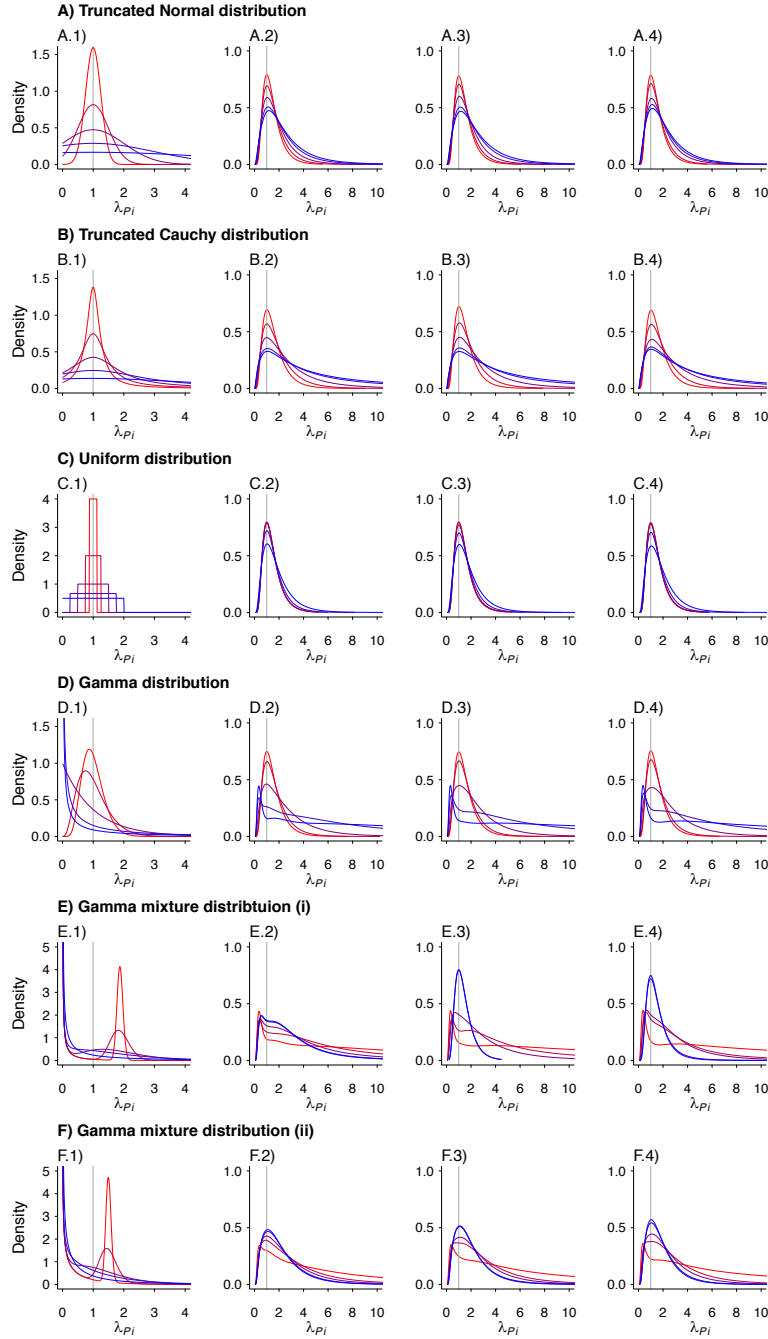


FIGURE S5: Estimated Density Kernels For Random Effects Model. We plot the estimated density kernels for the individual effects estimated by the random effects model for each individual in a simulated study. In each row, column 1 is a reproduction of the densities used to produce the individual λ_{pi_j} . Columns 2-4 are the estimated density kernels for $\theta = 0.2$, $\theta = 0.5$, $\theta = 0.8$, respectively. Values are plotted on a red-to-blue color ramp corresponding to the parameterizations I-V, respectively, in order of least (I; red) to greatest (V; blue) variance as detailed in Table 4. **A)** Truncated Normal distribution; **B)** Truncated Cauchy distribution; **C)** Uniform distribution; **D)** Gamma distribution; **E)** Mixture of Gamma distributions (i) with means at 0.125 and 1.875; and **F)** Mixture of Gamma distributions (ii) with means at 0.5 and 1.5.

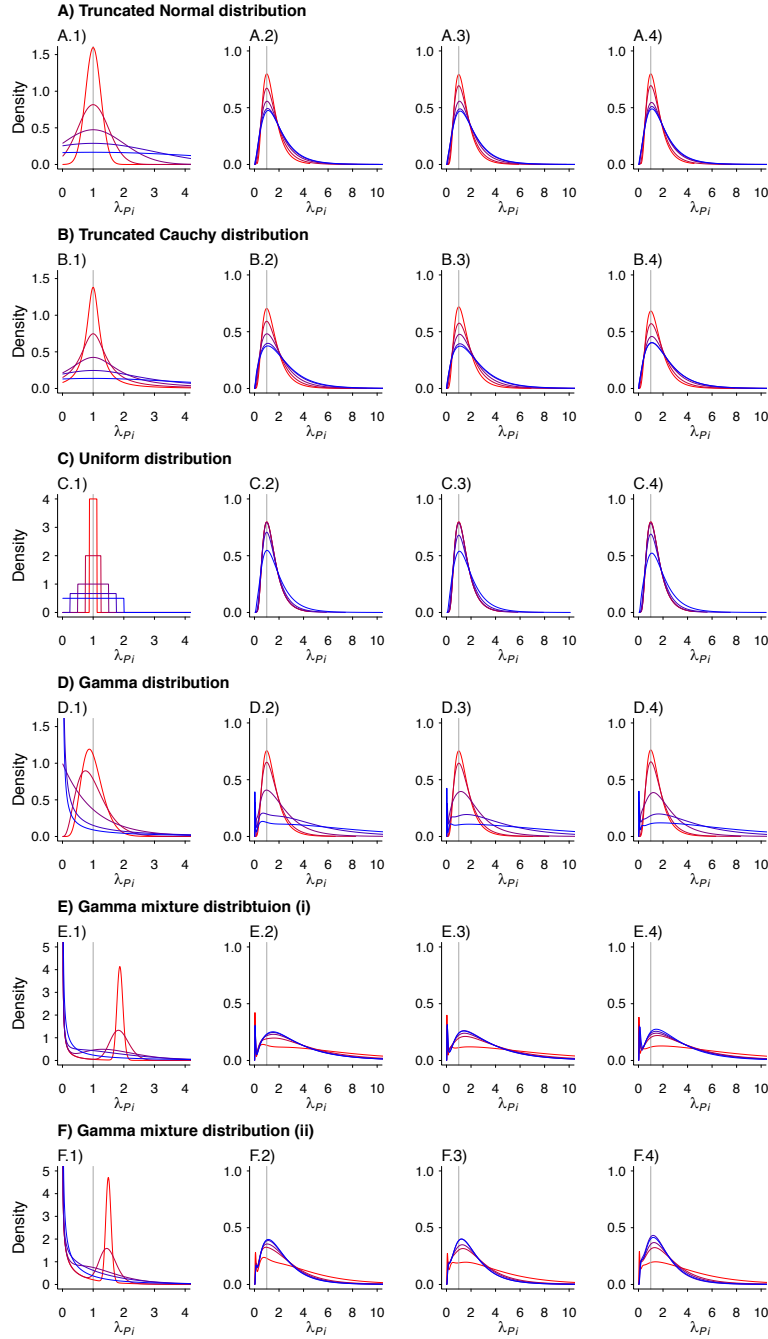


FIGURE S6: Estimated Density Kernels For Gamma Frailty Model. We plot the estimated density kernels for the individual effects estimated by the gamma frailty model for each individual in a simulated study. In each row, column 1 is a reproduction of the densities used to produce the individual λ_{p_i} . Columns 2-4 are the estimated density kernels for $\theta = 0.2$, $\theta = 0.5$, $\theta = 0.8$, respectively. Values are plotted on a red-to-blue color ramp corresponding to the parameterizations I-V, respectively, in order of least (I; red) to greatest (V; blue) variance as detailed in Table 4. **A)** Truncated Normal distribution; **B)** Truncated Cauchy distribution; **C)** Uniform distribution; **D)** Gamma distribution; **E)** Mixture of Gamma distributions (i) with means at 0.125 and 1.875; and **F)** Mixture of Gamma distributions (ii) with means at 0.5 and 1.5.

Supplemental Text: Description of Rotavirus Birth Cohort Design

Pregnant mothers were enrolled prior to childbirth, and children were followed from birth to ages 2 years (in Mexico City) and 3 years (in Vellore). Investigators aimed to identify all rotavirus infections through routine testing of asymptomatic stool specimens (collected by field workers at regular home visits) for rotavirus, and by monitoring children for anti-rotavirus seroconversion over serial blood draws at scheduled intervals. Active surveillance was undertaken for all cases of gastroenteritis among children to characterize symptoms and test diarrheal stool specimens for rotavirus.