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Appendix: Deduction process of the mathematical algorithm for calculation of ante-inclination

It is assumed that there is no coronal tilting and axial rotation of the pelvis in the current algorithm, which is an approximation of the clinical situation.

In the current study, we defined a coordinate system based on the APP plane of the pelvis: the mid-point of the bilateral anterior superior iliac spine was defined as the origin point O (0, 0, 0) of pelvic coordinate system; the X-axis was defined as the left-right direction**(leftwards as positive)**, the Y-axis was defined as the anterior-posterior direction **(posterior-wards as positive)**, and the Z-axis was defined as the cranial-caudal direction **(upwards as positive) (Appendix Fig. 1A)**. **V_1 is defined as the normal vector of the cup opening plane with the initial orientation pointing to the negative side of Z-axis: $V_1 = (0, 0, -1)^T$(Appendix Fig. 1A). Firstly, we rotate V_1 around the X-axis of the value of RA (radiographic anteversion) angle to get V_2. M_1 is the matrix of this rotation. (Appendix Fig. 1B)**

$$V_2 = M_1 * V_1$$

$$M_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(RA) & -\sin(RA) \\ 0 & \sin(RA) & \cos(RA) \end{bmatrix}$$

Secondly, we rotate V_2 around the Y-axis of the value of RI angle to get V_3. M_2 is the matrix of this rotation. (Appendix Fig. 1C)

$$V_3 = M_2 * V_2$$

$$M_2 = \begin{bmatrix} \cos(-RI * d) & 0 & \sin(-RI * d) \\ 0 & 1 & 0 \\ -\sin(-RI * d) & 0 & \cos(-RI * d) \end{bmatrix}$$

d is a bool variable represent the surgery side.

$$\mathrm{d} = \begin{cases} 1, & right\ side\ surgery \\ -1, & left\ side\ surgery \end{cases}$$

Thirdly, we rotate V_3 around the X-axis of the value of the PT angle to get V_4. M_3 is the matrix of this rotation. (Appendix Fig. 1D)

$$V_4 = M_3 * V_3$$

$$M_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\mathrm{PT}) & -\sin(PT) \\ 0 & \sin(\mathrm{PT}) & \cos(PT) \end{bmatrix}$$

Vp is defined as a "tool" vector for calculation of projection in the sagittal plane (the YOZ plane). (Fig. 1D)

$$V_p = (0, 1, 1)^T$$

Then V_5 is defined as the projection of V_4 on to the YOZ plane, and calculated

as the Hadamard product of V_4 and Vp, (Fig. 1D)

$$V_5 = V_4 \odot Vp$$

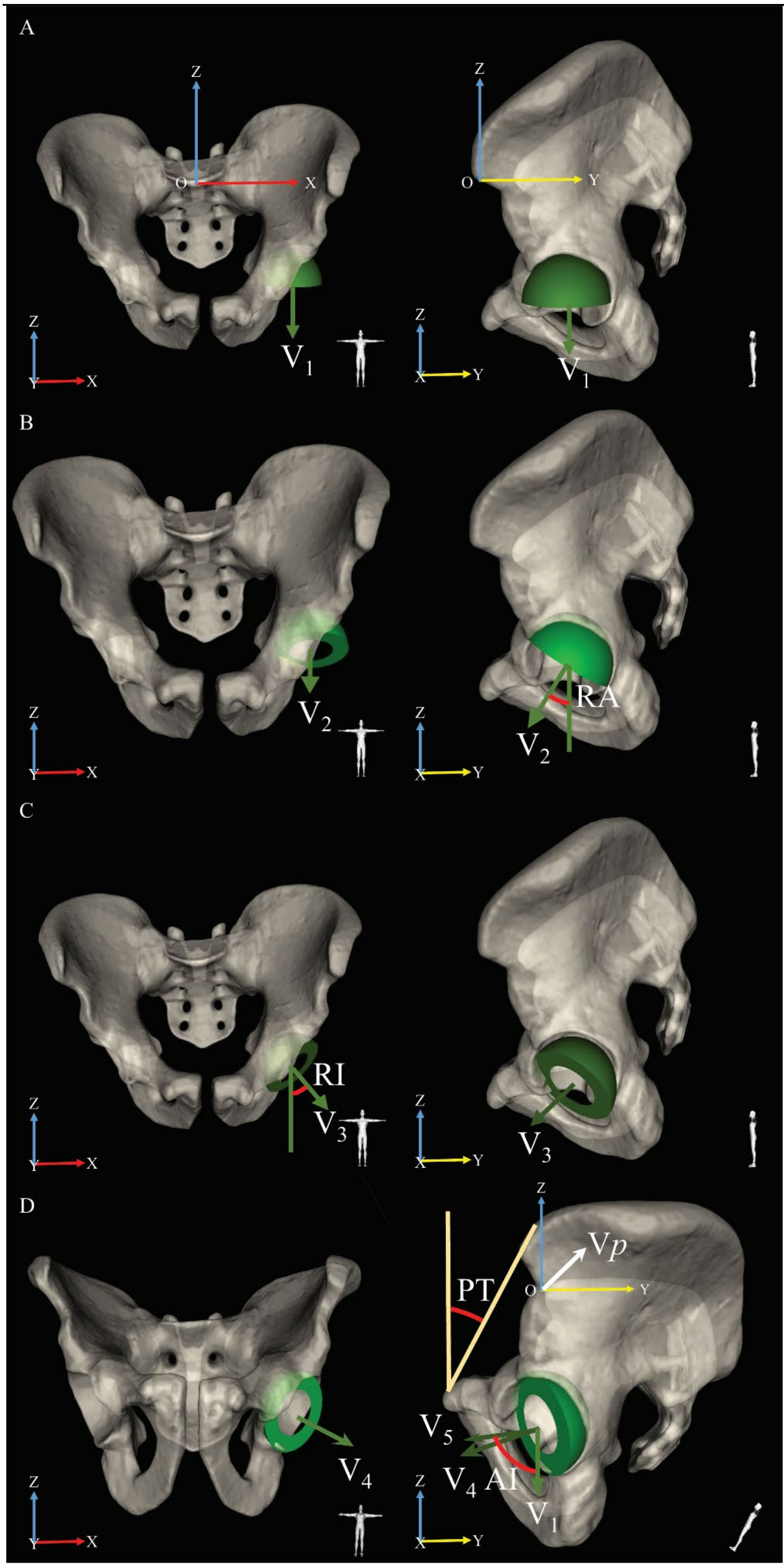
AI can be calculated as the angle between V_1 and V_5, (Appendix Fig. 1D)

$$AI = \mathrm{acos}\left(\frac{dot(V_1, V_5)}{\|V_1\| * \|V_5\|}\right)$$

Here, $dot(V_1, V_5)$ is the inner product of V_1 and V_5, $\|V_1\|$ is the L$_2$-norm of V_1, and $\|V_5\|$ is the L$_2$-norm of V_5.

As a result, AI can be calculated by

$$AI = \mathrm{acos}\left(\frac{dot(V_1, (M_3 * M_2 * M_1 * V_1) \odot V_p)}{\|V_1\| * \|(M_3 * M_2 * M_1 * V_1) \odot V_p\|}\right)$$


A
Z
O
X
Y
V1
B
V2
RA
C
RI
V3
D
V4
PT
Vp
V5
AI
V1

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Appendix Fig. 1 Schematic drawings illustrating the algorithm for deducing AI from RA, RI and PT. (A) The coordinate system (XYZ) of the pelvis is defined with the midpoint of the bilateral anterior superior iliac spine as the original point (point O), and V_1 was defined as the initial normal vector of the cup opening plane pointing to the negative Z axis, (B) the cup's normal vector turns to be V_2 after the first rotation of an angle of RA around the X-axis, (C) the second rotation of an angle of RI around the Y-axis of the pelvis rotates the normal vector to V_3, (D) the third rotation of the pelvis of an angle of PT around the X-axis of the pelvis leads to the resultant normal vector V_4, which projects onto the YOZ plane to be V_5, and the final AI angle was calculated between V_1.and V_5.